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SOLUTIONS

OF THE MORE DIFFICULT EXERCISES

AND

EXAMINATION PAPERS

IN THE

Canadian Edition

OF

HAMBLIN SMITH'S ARITHMETIC;

BY

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PREFACE.

This work has been prepared for the use of teachers and private students. With the multiplicity of duties ordinarily devolving on the teacher, time cannot always be had to solve all questions that may be presented by the pupil. Hence a work such as this becomes a great convenience, if not an actual necessity.

It has not been thought best to solve such exercises as are comparatively easy or merely mechanical. Hence those under the Simple Rules, many of those in Fractions, Extraction of Roots, the Compound Rules, Interest, etc., are omitted. The Examination Papers have all been solved.

The solutions have been given with strict reference to the Unitary Method, thus showing its applicability to questions of every variety and every degree of difficulty. They do not exhibit all the calculations at large, but they always furnish results which serve to verify the operations at the successive stages of the process. In this way all that is necessary has been brought within a narrow compass, and the connection of the different parts of each solution will be more readily perceived.

It has not been the aim of the Authors to make a mere Key, but to exhibit the best and neatest mode of working Arithmetical Exercises. Not only are neatness and method encouraged by the habit of arranging figures in their exact places, but the accuracy of the answer is best secured by the same means.

Indications of any errors or obscurities will be thankfully received.

Toronto, May, 1879,

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SOLUTIONS

OF THE MORE DIFFICULT EXAMPLES IN THE CANADIAN EDITION

OF

HAMBLIN SMITH'S ARITHMETIC.

EXAMINATION PAPERS.

VI.—Page 41.

2. $$\begin{array}{r} 768947\,54 \\ (112)(7)(56) \\ \hline 538263278 \\ 4306106224 \\ 8612212448 \\ \hline 8670344882024 \end{array}$$

See art. 50.

3. $(8376 + 5684) \times (8376 - 5684) \div 7859 = 4816$ and rem. 576.

$$7859 - 576 = 7283.$$

5. If the sum of two numbers is added to the difference of the two numbers, the result is equal to twice the greater number.

$$\therefore \text{ the greater number} = \frac{2 \times 4331 + 2 \times 3353}{2} = 7684.$$

VII.—Page 41.

1. Cost of 1 a. $= \dfrac{8 \times \$23 + 7 \times \$89}{3} = \$269.$

2. Time for 19 men $= \dfrac{18 \times 76}{19}$ da. $= 72$ da.

3. Number of each $= \frac{6300}{3.50+21.50} = 252.$

4. Number $= 375 \times 789 + 362 = 296237.$

5. $19 + 17 + 15 = 51.$

$\therefore$ sum received by first out of \$51 $=$ \$19.

" " \$35700 $= \frac{35700 \times \$19}{51}$

$= 700 \times \$19$

$= \$13300.$

" second $= \frac{35700 \times \$17}{51}$

$= \$11900.$

" third $= \frac{35700 \times \$15}{51}$

$= \$10500.$

VIII.—Page 42.

1. In working such problems as this, begin with the last result and work towards the beginning of the example, always reversing the operation described in the problem.

Arith. Comp. of $7846 = 10000 - 7846 = 2154.$ Art. 49.

" " $3479 = 10000 - 3479 = 6521$

Difference $= 4367$

Number $= [\{(883 + 4367) \div 10\} + 1225] \times 4$

$= 7000.$

2. 5 lbs. tea $= 15$ lbs. coffee $= \frac{15 \times 8}{4}$ lbs. sugar.

$\therefore 75$ lbs. tea $= \frac{15 \times 15 \times 8}{4}$ lbs. sugar $= 450$ lbs.

3. Number $= 13675 + (45209 - 27645) = 31239.$

4. 9 times the value of a saddle $= \$261$;

$\therefore$ value of saddle $= \$\frac{261}{9} = \$29$

and value of horse $= 8 \times \$29 = \$232.$

5. Cost of cattle per head $= \$18 + \$2 = \$20.$

Number bought $= \frac{6400}{20} = 320.$

" sold $= \frac{3600}{18} = 200.$

Hence the difference, 120 cattle, must sell for $(\$6400 - \$3600 + \$800) = \$3600$;

$\therefore$ selling price of 1 head $= \frac{\$3600}{120} = \$30.$

IX.—Page 42.

1. See art. 51.
2. Number $= 99995 \times 99995 = 9999000025.$
3. Share of youngest son $= \$1789.$
" second " $= 5 \times \$1789.$
" eldest " $= 15 \times \$1789.$

$\therefore$ value of property $= \$(15 \times 1789 + 5 \times 1789 + 1789)$
$= \$(21 \times 1789)$
$= \$37569.$

4. Number of steps $= \frac{4}{3} \times \frac{17694}{2} = 11796.$

5. Indebtedness $= \frac{7770 \times \$100}{37} = \$21000.$

Sum due creditor $= \frac{1998 \times \$100}{37} = \$5400.$

Paper X.—Page 43.

1. See art. 47.

2. Use (1728) (144) (12) as the multiplier and multiply by 12; then multiply this product by 12 and the

new product by 12; add the three partial products together as in art. 50.

3. Writing the arithmetical complements of the subtractive quantities, we have

1
$\bar{1}8$
4
$\bar{1}2$
16
$\bar{1}68$
64
$\bar{1}872$
256
$\bar{1}488$
1024
$\bar{1}7952$
4096
$\bar{1}1808$
16384
$\bar{1}67232$
65536
$\bar{1}868928$
262144
$\bar{1}475712$
1048576
$\bar{1}7902848$
4194304

2796203

4.

786464	3457
78	6464
	78

Quotient = 786543

See art. 51.

5. 36 times remainder = quotient.

6 times remainder = divisor.

$\therefore$ 43 times remainder = 516;

$$\therefore \text{remainder} = \frac{516}{43} = 12.$$

$\therefore$ Dividend $= 12 + 72 \times 432 = 31116.$

Highest Common Factor.

Examples (xxiv). Page 46.

The following rule will be found much easier in practice than the one given in the text book.

Divide all the given numbers by the least of them, and bring down the remainders.

2. Divide the first divisor and all of the first remainders by the least of them, and bring down the remainders.

3. Proceed in this manner until a remainder is found that will divide all the other remainders, and the divisor last used, and this will be the highest common factor required.

3. 365, 511, 803.
365, 146, 73.

We divide by 365, writing down the remainders 146 and 73. 73 will divide the first divisor, 365, and the other remainders, and is therefore the H. C. F.

4. 232, 290, 493.
232, 58, 29. H. C. F. is 29.

5. 492, 1476 1763.
492, 0 287.
205, 287.
205, 82.
41, 82. H. C. F. is 41.

6. 148, 444, 592, 703.
148, 0, 0, 111.
37, 111.
H. C. F. is 37.

I.—Page 49.

1. Number = (L. C. M. of 13, 15 and 17) + 12.
= 3315 + 12 = 3327.

2. L. C. M. of 33, 27 and 30 = 2970.

$$\text{Number of times} = \frac{103950}{2970} = 35.$$

3. Length of rail = H. C. F. of 23023 ft. and 17765 ft. = 11 ft.

$$\text{Number of rails} = 6 \times \frac{2 \times 23023 + 2 \times 17765}{11}$$
$$= 44496.$$

4. Since H. C. F. of 210 and 330 = 30, ∴ 11 revolutions of small wheel = 7 revolutions of large one.

5. The prime factors of 2772 = 2, 2, 3, 3, 7 and 11. The required numbers must be divisible by 12, and have their L. C. M. 2 × 2 × 3 × 3 × 7 × 11.

∴ one number = 12 × 3 = 36.
" a second = 12 × 7 = 84.
" a third = 12 × 11 = 132.

II.

2. We must here find the 3 smallest and also the 3 largest numbers that will exactly divide 600.

The prime factors of 600 = 2, 2, 2, 3, 5 and 5.

∴ the 3 smallest bags must hold 1 bu., 2 bu., or 3 bu., and the 3 largest bins, 300 bu., 200 bu., or 150 bu.

3. The L. C. M. of 5, 22 and 75 = 22 × 75 = 1650.

∴ smallest sum = $1650.

4. Time required by first horse to go once round

$$= \frac{5280}{440} \text{ min.} = 12 \text{ min.}$$

Time required by second horse to go once round

$$= \frac{5280}{352} \text{ min.} = 15 \text{ min.}$$

Time required by third horse to go once round

$$= \frac{5280}{264} \text{ min.} = 20 \text{ min.}$$

Time required = L. C. M. of 12 min., 15 min., and 20 min. = 60 min.

5. Number = (L. C. M. of 675, 1050, and 4368) + 32
= 982800 + 32 = 982832.

III —Page 50.

1. Resolve the number into its prime factors. Form as many series as there are different prime factors, making 1 the first term of each series; the first power of the prime factor the second term; the second power of that factor the third term, &c. Multiply these series together.

Prime factors of 8100 = 2, 2, 3, 3, 3, 3, 5 and 5.

1st series = 1, 2, 4.

2nd " = 1, 3, 9, 27, 81.

3rd " = 1, 5, 25.

1, 3, 9, 27, 81

1, 2, 4

1, 3, 9, 27, 81, 2, 6, 18, 54, 162, 4, 12, 36, 108, 324.

1, 5, 25

1, 3, 9, 27, 81, 2, 6, 18, 54, 162, 4, 12, 36, 108, 324.
5, 15, 45, 135, 405, 10, 30, 90, 270, 810, 20, 60, 180.
540, 1620, 25, 75, 225, 675, 2025, 50, 150.
450, 1350, 4050, 100, 300, 900, 2700, 8100.

2. The prime factors of $10440 = 2^3, 3^2, 5$ and 29.
$\therefore$ number required $= 29$.

3. See art. 37.

4. Time required by $A = 12$ hrs ; $B = 15$ hrs; $C = 20$ hrs.
L. C. M. of 12, 15 and $20 = 60$.
$\therefore$ time required $= 60$ hrs.
$\therefore$ distance walked by $A = 60 \times 5$ mi. $= 300$ mi.
" " $B = 60 \times 4$ mi. $= 240$ mi.
" " $C = 60 \times 3$ mi. $= 180$ mi.

5. Number of grs. in 1 lb. Avoir. $= \dfrac{175 \times 5760}{144} = 7000$.
Number of grains required $=$ H. C. F. of 5760 and 7000.
$= 40$.

IV.—Page 50.

2. Number required $= \dfrac{1270374}{2 \times 3129} = 203$.

3. Distance gone $= (360 \times 11 \times 13)$ feet.
$= \dfrac{360 \times 11 \times 13}{5280}$ mi. $= 9\frac{3}{4}$ mi.

4. Number of holes to furnish a day's work for all together $= 36 + 32 + 30 = 98$.
Number required $=$ L. C. M. of 36, 32, 30 and 98
$= 70560$.

5. Cost of sugar $= 14 \times 276 \times 8$ cents.
Cost of 1 firkin $= 56 \times 23$ cents.
No. of firkins $= \dfrac{14 \times 276 \times 8}{56 \times 23} = 24$.

V.—Page 50.

2. We are required to find the H. C. F. of $(10974 - 54)$ and $(15336 - 36)$.

The H. C. F. of 10920 and 15300 = 60.

3. Since 2 is in the units' place the remainder = 3. So that the subtraction may be completed 1 must be borrowed from the 7 in the millions' place, thus the remainder in the millions' place = 6.

4. Length of avenue = 3×5280 ft. = 15840 ft.

L. C. M. of 6, 8, 9, 10 and 12 = 360.

Number of times there are 5 trees in a row = $\frac{3 \times 5280}{360}$

$= 44.$

Total number of trees = $\frac{15840}{6} + \frac{15840}{8} + \frac{15840}{9} + \frac{15840}{10} + \frac{15840}{12} = 9284$

5. Number $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5 \times 11 \times 17$

$= 3366000.$

Subtraction of Fractions.

Page 57.

The following method will often be found much simpler than the rule given in the text-book:

Let $\frac{a}{b}$ and $\frac{c}{d}$ be the fractions;

then $\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd} = \frac{a(d-c) - c(b-a)}{bd.}$

The advantage of this method will be great when the terms of the fractions are large numbers and nearly equal to each other.

Examples (xxxi). Page 57.

3. $\frac{12}{13} - \frac{11}{12} = \frac{12\,(12 - 11) - 11\,(13 - 12)}{13 \times 12} = \frac{1}{156}.$

8. $\frac{359}{360} - \frac{199}{200} = \frac{359 \times 1 - 199 \times 1}{360 \times 200} = \frac{160}{360 \times 200}$
$= \frac{1}{450}$.

Complex Fractions.

Examples (xxxviii). Page 67.

7. $\cfrac{2}{5 + \cfrac{6}{9 + \frac{3}{4}}} = \cfrac{2}{5 + \cfrac{24}{36 + 3}} = \frac{2}{5 + \frac{8}{13}} = \frac{26}{73}$.

9. $\cfrac{5}{2 - \cfrac{1}{4 - \frac{2}{5}}} = \cfrac{5}{2 - \cfrac{5}{20 - 2}} = \frac{90}{36 - 5} = 2\frac{28}{31}$.

10. $\cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{2}{5}}}} = \cfrac{1}{1 + \cfrac{1}{1 + \frac{5}{7}}} = \frac{1}{1 + \frac{7}{12}} = \frac{12}{19}$.

Examples (xxxix.) Page 68.

1. $3\frac{2}{5} \div (2\frac{1}{3} + 1\frac{5}{7}) = 3\frac{2}{5} \div (2\frac{7}{21} + 1\frac{15}{21})$
$= 3\frac{2}{5} \div 4\frac{1}{21}$
$= \frac{17}{5} \times \frac{21}{85}$
$= \frac{21}{25}$.

2. $(4\frac{3}{11} + 2\frac{1}{5}) \div 35\frac{3}{5} = (4\frac{15}{55} + 2\frac{11}{55}) \div 35\frac{3}{5}$
$= \frac{356}{55} \times \frac{5}{178}$
$= \frac{2}{11}$.

7. $\frac{2}{3}$ of $\frac{5}{9} + \frac{3}{7} \div \frac{4}{5} = \frac{10}{27} + \frac{3}{7} \times \frac{5}{4}$
$= 5\left(\frac{2}{27} + \frac{3}{28}\right)$
$= 5\left(\frac{56 + 81}{27 \times 28}\right)$
$= \frac{685}{756}$.

When the num. or den. has a common factor, it should be taken out, the operations performed, and the common factor introduced at the last.

8. $(1\frac{1}{13} \div \frac{2}{7})$ of $7\frac{7}{12} - 1\frac{3}{5} = \frac{14}{13} \times \frac{7}{2} \times \frac{91}{12} - 1\frac{3}{5} = 22\frac{14}{24} - 1\frac{3}{5}$
$= 20\frac{103}{120}$.

9. $(\frac{4}{9} - \frac{3}{11})\ (2\frac{3}{4} + 3\frac{2}{3}) = \frac{17}{9 \times 11} \times \frac{77}{12} = \frac{17}{9} \times \frac{7}{12} = 1\frac{11}{108}$

11. $\frac{(2 + \frac{1}{5}) \div (3 + \frac{1}{7})}{(\frac{1}{2} - \frac{1}{3}) \times (4 - 3\frac{3}{7})} = \frac{\frac{11}{5} \times \frac{7}{22}}{\frac{1}{6} \times \frac{4}{7}} = \frac{1}{5} \times \frac{7}{2} \times \frac{3}{1} \times \frac{7}{2} = 7\frac{7}{20}$.

12. $\frac{(3\frac{1}{3} - 2\frac{1}{2}) \div \frac{5}{6} \text{ of } \frac{3}{8}}{2\frac{2}{3} \div (\frac{1}{2} + \frac{1}{4})} = \frac{\frac{5}{6} \div \frac{5}{16}}{\frac{8}{3} \times \frac{4}{3}} = \frac{5}{6} \times \frac{16}{5} \times \frac{3}{8} \times \frac{3}{4} = \frac{3}{4}$.

Note. Two or more fractions connected by *of* are always considered as *one* quality.

Miscellaneous Examples in Fractions.

Examples (xl). Page 69.

5. $3\frac{2}{5} \times 3\frac{3}{7} \div (1\frac{5}{7} \times 1\frac{13}{21}) = \frac{17}{5} \times \frac{24}{7} \times \frac{7}{12} \times \frac{21}{34} = \frac{21}{5} = 4\frac{1}{5}$;

Note.—*Indicate* all operations before performing any of them. It is much easier to simplify *before* performing the multiplication or division than *after*.

8. $(\frac{1}{3} + \frac{4}{7})\ \frac{20\frac{1}{4}}{3\frac{6}{7} + 2\frac{1}{4}} = \frac{19}{21} \times \frac{81}{15\frac{3}{7} + 9} = \frac{19}{21} \times \frac{27}{5\frac{1}{7} + 3} = \frac{19}{7} \times \frac{63}{36 + 21} = 3$.

9. $(3\frac{4}{5} + 5\frac{1}{9} - \frac{1}{45})\ (4\frac{1}{5} - 3\frac{1}{4}) = 8\frac{8}{9} \times \frac{19}{20} = \frac{4 \times 19}{9}$,
and $1\frac{5}{11} + 2\frac{1}{8} - (2\frac{9}{16} - \frac{1}{8} - \frac{1}{22}) = 3\frac{51}{88} - (2\frac{7}{16} - \frac{1}{22})$
$= \frac{19}{16}$.

$\therefore$ quotient $= \frac{4 \times 19}{9} \div \frac{19}{16} = \frac{64}{9} = 7\frac{1}{9}$.

10. $(1\frac{1}{3} + 2\frac{2}{7})\ \left(\frac{5\frac{1}{16}}{4\frac{6}{7} + 1\frac{1}{4}}\right) = \frac{76}{21} \times \frac{81}{77\frac{5}{7} + 20} = \frac{76}{1} \times \frac{27}{544 + 140} = 3$.

11. $(7\frac{1}{5} + 1\frac{4}{5} - \frac{1}{45})\ (2\frac{1}{4} - \frac{4}{5}) = 8\frac{40}{45} \times \frac{29}{20} = \frac{4 \times 29}{9}$

and $4\frac{1}{8} - \frac{6}{11} - (2\frac{7}{8} - \frac{7}{16} - \frac{1}{22}) = 3\frac{51}{88} - 2\frac{69}{176} = 1\frac{33}{176}$

$= \frac{12}{16}$.

$\therefore$ quotient $= \frac{4 \times 29}{9} \div \frac{12}{16} = \frac{1856}{171} = 10\frac{146}{171}$.

12. $\frac{6\frac{3}{4} - 1\frac{5}{14}}{2\frac{1}{6} + 1\frac{3}{7}} = \frac{94\frac{1}{2} - 19}{30\frac{1}{3} + 20} = \frac{75\frac{1}{2}}{50\frac{1}{3}} = \frac{453}{302} = 1\frac{1}{2}$.

$(\frac{5}{7}$ of $1\frac{6}{13}) \div \frac{25}{3\frac{1}{4}} = \frac{5}{7} \times \frac{19}{13} \times \frac{13}{4} \times \frac{7}{25} = 1\frac{1}{5}$.

13.

$$\frac{1}{4 - \dfrac{1}{2 - \dfrac{1}{1 - \frac{5}{13}}}} = \frac{1}{4 - \dfrac{1}{2 - \dfrac{13}{13 - 5}}} = \frac{1}{4 - \dfrac{8}{16 - 13}}$$

$= \frac{3}{4}$.

$$\frac{1}{4 + \dfrac{1}{1 - \dfrac{1}{2 - \frac{9}{16}}}} = \frac{1}{4 + \dfrac{1}{1 - \dfrac{16}{32 - 9}}} = \frac{1}{4 + \dfrac{23}{23 - 16}}$$

$$= \frac{7}{28 + 23} = \frac{7}{51}.$$

14. $\frac{10\frac{2}{5} - 1\frac{5}{7}}{7\frac{1}{8} + 3\frac{3}{40}} = \frac{8\frac{24}{35}}{10\frac{1}{5}} = \frac{304}{35} \times \frac{5}{51} = \frac{304}{357}$.

$(\frac{3}{7}$ of $2\frac{1}{17}) \div \frac{1\frac{2}{3}}{2\frac{3}{7}} = \frac{3}{7} \times \frac{35}{17} \times \frac{3}{5} \times \frac{17}{7} = 1\frac{2}{7}$.

15. $\frac{8\frac{7}{8} - 7\frac{6}{7} + 5\frac{5}{6} - 4\frac{4}{5}}{9\frac{9}{10} - 8\frac{13}{15} + 7\frac{7}{8} - 6\frac{6}{7}} = \frac{1\frac{1}{56} + 1\frac{1}{30}}{1\frac{1}{30} + 1\frac{1}{56}} = 1$.

$1\frac{21}{43} \times \frac{37787}{75008} - \frac{64}{43} \times \frac{37787}{75008} = \frac{8709}{11672} = \frac{3}{4}$.

16. $\frac{5 - \dfrac{1}{5 - \frac{1}{5}}}{3 - \dfrac{1}{3 - \frac{1}{3}}} \times \frac{9}{23}$ of $7 = \frac{5 - \frac{5}{24}}{3 - \frac{3}{8}} \times \frac{9}{23} \times 7$

$= 5 \times \frac{23}{24} \times \frac{1}{3} \times \frac{8}{7} \times \frac{9}{23} \times 7 = 5$.

$$\frac{6+\cfrac{1}{6-\frac{1}{6}}}{4-\cfrac{1}{4-\frac{1}{4}}}\times 10\tfrac{8}{9}=\frac{6+\frac{6}{35}}{4-\frac{4}{15}}\times\frac{98}{9}$$

$$=6\times\tfrac{36}{35}\times\tfrac{1}{4}\times\tfrac{15}{14}\times\tfrac{98}{9}=18.$$

17. $$\frac{8\frac{3}{5}-7\frac{3}{4}+5\frac{2}{3}-4\frac{1}{2}}{13-11\frac{9}{10}+10\frac{7}{9}-9\frac{17}{20}}=\frac{2+\frac{3}{5}+\frac{2}{3}-\frac{3}{4}-\frac{1}{2}}{3+\frac{7}{9}-\frac{9}{10}-\frac{17}{20}}$$

$$=\frac{\frac{121}{60}}{\frac{365}{180}}=\frac{121\times 3}{365};$$

then $\frac{121\times 3}{365}\times\frac{2}{11}\times 365=66.$

18. $$\frac{\frac{1}{21}\times 5\frac{17}{23}\times 6\frac{3}{11}+6\frac{19}{51}\times 1\frac{23}{49}\div 2\frac{5}{17}+1\frac{10}{49}}{9\frac{16}{57}\times 1\frac{22}{23}\div 5\frac{17}{38}+3\frac{11}{78}\times 6\frac{17}{21}\div 7\frac{21}{32}}\times 12\tfrac{4}{9}$$

$$=\frac{\frac{1}{21}\times\frac{132}{23}\times\frac{69}{11}+\frac{325}{51}\times\frac{72}{49}\times\frac{17}{39}+\frac{59}{49}}{\frac{529}{57}\times\frac{45}{23}\times\frac{38}{207}+\frac{245}{78}\times\frac{143}{21}\times\frac{32}{245}}\times\frac{112}{9}$$

$$=\frac{\frac{12}{7}+\frac{200}{49}+\frac{59}{49}}{\frac{10}{3}+\frac{176}{63}}\times\frac{112}{9}=\frac{\frac{343}{49}}{\frac{386}{63}}\times\frac{112}{9}$$

$$=\tfrac{343}{49}\times\tfrac{63}{386}\times\tfrac{112}{9}=\tfrac{2744}{193}=14\tfrac{42}{193}.$$

19. $$\frac{\frac{1}{23}\text{ of }6\frac{13}{17}\text{ of }24\frac{11}{13}-4\frac{13}{18}\times 3\frac{33}{34}\div 3\frac{37}{96}}{8\frac{17}{19}\times 5\frac{14}{39}\div 4\frac{15}{32}-7\frac{19}{20}\times 5\frac{11}{65}\div 14\frac{21}{25}}\times 4\tfrac{8}{23}$$

$$=\frac{\frac{1}{23}\times\frac{115}{17}\times\frac{323}{13}-\frac{85}{18}\times\frac{135}{34}\times\frac{96}{325}}{\frac{169}{19}\times\frac{209}{39}\times\frac{32}{143}-\frac{159}{20}\times\frac{336}{65}\times\frac{25}{371}}\times\frac{100}{23}$$

$$=\frac{\frac{95}{18}-\frac{72}{13}}{\frac{32}{3}-\frac{36}{13}}\times\frac{100}{23}=\tfrac{1}{13}\times 23\times 39\times\tfrac{1}{308}\times\tfrac{100}{23}=\tfrac{75}{77}.$$

20. $$\frac{19}{7\times\cfrac{2}{3-1\frac{2}{3}}}\times\frac{7735}{67184}\div(1\tfrac{3}{16}-\tfrac{47}{48})$$

$$=\frac{19}{7\times\frac{6}{4}}\times\tfrac{7735}{67184}\times\tfrac{48}{10}=\tfrac{19}{7}\times\tfrac{4}{6}\times\tfrac{7735}{67184}\times\tfrac{48}{10}=1.$$

21. $$\frac{1}{2+\cfrac{3}{4+\frac{5}{6}}}\times\tfrac{4862}{4147}\div(1\tfrac{1}{2}-\tfrac{23}{38})=\tfrac{29}{76}\times\tfrac{4862}{4147}\times\tfrac{38}{34}=\tfrac{1}{2}.$$

22. $\dfrac{\frac{7}{4-\frac{5}{6}}-\frac{5}{6-\frac{3}{8}}}{\frac{4}{7-\frac{4}{7}}+\frac{2}{4-\frac{2}{5}}} \times \dfrac{\frac{1}{\frac{1}{2}-\frac{27}{59}}-13}{19-\frac{1}{\frac{1}{2}-\frac{6}{31}}} = \dfrac{\frac{42}{10}-\frac{40}{45}}{\frac{28}{45}+\frac{10}{18}} \times \dfrac{\frac{118}{5}-13}{19-\frac{62}{19}}$

$= \dfrac{\frac{1130}{19\times45}}{\frac{106}{90}} \times \dfrac{\frac{53}{5}}{\frac{299}{19}} = \frac{1130}{19} \times \frac{1}{45} \times \frac{90}{106} \times \frac{53}{5} \times \frac{19}{299}$

$= \frac{5\times226}{19} \times \frac{1}{45} \times \frac{45}{53} \times \frac{53}{5} \times \frac{19}{299} = \frac{226}{299}.$

23. $\dfrac{\frac{10}{11}+\frac{18}{19}}{\frac{12}{7}-\frac{5}{11}} \times \dfrac{\frac{9}{4}-\frac{22}{14}}{\frac{56}{57}-\frac{110}{228}} = \dfrac{\frac{388}{11\times19}}{\frac{97}{7\times11}} \times \dfrac{\frac{19}{28}}{\frac{114}{228}}$

$= \frac{388}{11} \times \frac{1}{19} \times \frac{77}{97} \times \frac{19}{28} \times \frac{228}{114} = 2.$

EXAMINATION PAPERS.

I.—Page 71.

2. $18\frac{3}{4} \times \$2\frac{2}{5} + 27\frac{1}{2} \times \$\frac{3}{20} = \frac{75}{4} \times \$\frac{12}{5} + \frac{55}{2} \times \$\frac{3}{20}$
$= \frac{15}{4} \times \$12 + \frac{11}{2} \times \$\frac{3}{4}$
$= \$49\frac{1}{8}.$

3. Sum $= 12\frac{32}{40} + 8\frac{35}{40} = 21\frac{27}{40}.$
Diff. $= 12\frac{32}{40} - 8\frac{35}{40} = 3\frac{37}{40}.$
And $21\frac{27}{40} \div 3\frac{37}{40} = \frac{867}{40} \times \frac{40}{157} = 5\frac{82}{157}.$

4. $\frac{4}{7}$ of $\frac{5}{11}$ of share $= \$3600$;
$\therefore$ whole $= \frac{7}{4}$ of $\frac{11}{5}$ of $\$3600 = \$13860.$

5. Since the sum of two numbers added to their diff. = twice the greater, we have
$4\frac{1}{5} + 2\frac{4}{7} = 2\frac{37}{35} =$ twice the greater;
$\therefore$ greater $= 2\frac{37}{70} = 3\frac{27}{70}.$
And $4\frac{1}{5} - 3\frac{27}{70} = \frac{57}{35} =$ the less.

II.—Page 71.

1. The first number is to be made the numerator, and the second number the denominator of the same fraction.

$$\frac{3\frac{1}{2}}{9\frac{1}{5}} = \frac{35}{92}.$$

2. Art. 66. The relative magnitudes will be obvious when the fractions are reduced to the same denominator.

3. The sum of the fractions is $\frac{3}{11}$.

$\frac{3}{11} = \frac{3000}{11000} = \frac{2731+}{11000}$, next less than $\frac{2732}{11000}$;
And $1 - \frac{2732}{11000} = \frac{7268}{11000}$, the fraction required.

4. $\frac{2+5}{3+7} = \frac{7}{10} = \frac{21}{30}$, and $\frac{2}{3} = \frac{20}{30}$;

$\therefore \frac{2+5}{3+7}$ is greater than $\frac{2}{3}$.

Also $\frac{2+5}{3+7} = \frac{7}{10} = \frac{49}{70}$ and $\frac{5}{7} = \frac{50}{70}$;

$\therefore \frac{2+5}{3+7}$ is less than $\frac{5}{7}$.

5. $\frac{3}{8}$ of ship $= \frac{1}{4}$ of cargo
$\frac{1}{8}$ " $= \frac{1}{12}$ "
$\therefore$ ship $= \frac{2}{3}$ of cargo;
$\therefore \frac{2}{3}$ of cargo + $\frac{3}{5}$ of cargo $=$ \$60000
$\frac{5}{3}$ " $=$ \$60000;
$\therefore$ cargo $= \frac{3 \times 60000}{5} =$ \$36000.
Ship $=$ \$60000 $-$ \$36000 $=$ \$24000.

III

1. Art. 59. The denominator, *i.e.*, the "*name-giver*," because it gives the name to the parts.

The numerator, *i.e.*, the "*numberer*," or "*counter*," because it indicates how many of the parts named by the denominator are to be taken.

2. $\frac{4}{7}$ of $\frac{3}{8}$ of $2\frac{1}{2}$ bbls. $= \frac{6}{7}$ bbl.

Value of $\frac{6}{7}$ bbl. $= \$7\frac{1}{3}$;

" $\frac{1}{7}$ bbl. $= \frac{\$7\frac{1}{3}}{6} = \$1\frac{1}{9}$;

value of $\frac{7}{7}$ or 1 bbl. $= \$\frac{7 \times 11}{9}$;

$\therefore$ " $2\frac{2}{11}$ bbls. $= \$\frac{2\frac{2}{11} \times 7 \times 11}{9} = \$18\frac{2}{3}$.

3. $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$;

then $\frac{1}{2} = \frac{1+2+3+4+5+6}{2+4+6+8+10+12} = \frac{1}{2}$.

4. Sum of fractions $= \frac{77}{60}$; then $2 - \frac{77}{60} = \frac{43}{60}$;

and $\frac{3}{5}$ of $\frac{27}{40}$ of $88 \times \frac{43}{60} = \frac{1}{5} \times \frac{27}{20} \times 11 \times \frac{43}{5}$.

To find what fraction this product is of 999, we have

$$\frac{\frac{1}{5} \times \frac{27}{20} \times 11 \times \frac{43}{5}}{999} = \frac{4773}{186000}.$$

5. C's age is evidently 84 years.

B's " $= \frac{4}{7}$ of C's $= \frac{4}{7}$ of $84 = 48$ yrs.

A's " $= \frac{5}{12}$ of B's $= \frac{5}{12}$ of $48 = 20$ yrs.

IV.—Page 72.

1. In the operation of addition of integers, the addends must have the same name, in order that their sum may be expressed by one number; so also in fractions, the addends must have the same fractional unit in order that their sum may be expressed as one fraction.

2. $17\frac{1}{5}$ contains $3\frac{2}{7}$, 5 times, with remainder $\frac{27}{35}$; if, therefore, $\frac{27}{35}$ be taken from $17\frac{1}{5}$, the remainder will contain $3\frac{2}{7}$ an exact number of times, viz., 5 times.

3. If operations the reverse of those indicated in the question, be performed on $2\frac{3}{5}$, the required number will be found; hence,

$$\left\{(2\tfrac{3}{3}+\tfrac{1}{2}\text{ of }\tfrac{3}{7}\text{ of }14\tfrac{4}{9})\div\frac{2\frac{1}{3}}{3}-2\tfrac{3}{4}\right\}\times 8\tfrac{4}{17}$$

$$=(5\tfrac{1}{2}\tfrac{6}{1}\times\tfrac{6}{5}-2\tfrac{3}{4})\times 8\tfrac{4}{17}$$

$$=4\tfrac{2}{1}\tfrac{3}{4}\tfrac{3}{0}\times 8\tfrac{4}{17}$$

$$=34\tfrac{5}{17}.$$

4. Carriage $=\frac{7}{8}$ of horse;

$\therefore$ horse $+\frac{7}{8}$ of horse $=\$225$;

$\frac{15}{8}$ value of horse $=\$225$;

$\therefore$ value of horse $=\$\frac{8\times 2\,2\,5}{1\,5}=\$120.$

Carriage $=\$225-\$120=\$105.$

Harness $=\frac{24}{120}$ of $\$120=\$25.$

5. Let 1 represent B's share, then

since B's $=1$

A's $=3-\$88,$

and C's $=2-\$44+\$176;$

and A's $+B$'s $+C$'s $=6+\$44=\$8888.$

$\therefore\ 6=\$8844,$

$1=\$1474=B$'s share.

A's $=3\times\$1474-\$88=\$4334.$

C's $=2\times\$1474+\$132=\$3080.$

V.—Page 72.

1. Arts. 80 and 84.

2. $$\frac{3\frac{1}{3}\times 3\frac{1}{3}\times 3\frac{1}{3}-1}{3\frac{1}{3}\times 3\frac{1}{3}-1}=3\tfrac{1}{3}+\frac{2\frac{1}{3}}{3\frac{1}{3}\times 3\frac{1}{3}-1}$$

$$=3\tfrac{1}{3}+\frac{21}{10\times 10-9}$$

$$=3\tfrac{1}{3}+\tfrac{3}{1\,3}=3\tfrac{2}{3}\tfrac{3}{9}.$$

3. Smallest number equals the L. C. M. of $\$4\frac{1}{3}$, $\$5\frac{1}{6}$, and $\$2\frac{1}{2}$, which $=\$2015$, Art. 81; then

$$\frac{2015}{4\frac{1}{3}}=165\text{ sheep.}$$

$$\frac{2015}{5\frac{1}{6}} = 390 \text{ calves};$$

$$\frac{2015}{2\frac{1}{2}} = 806 \text{ pigs}.$$

4. After spending \$80 less than $\frac{3}{4}$ of his money John has $\frac{1}{3}$ of his money + \$80 left; then if 1 represent his money, we have

$$(\tfrac{1}{3}+\$80) - \{\tfrac{3}{7}(\tfrac{1}{3}+\$80)+\$40\} = \$40$$
$$\tfrac{4}{7}(\tfrac{1}{3}+\$80) - \$40 = \$40,$$
$$\text{or, } \tfrac{1}{7}(\tfrac{1}{3}+\$80) = \$20$$
$$\tfrac{1}{3}+\$80 = \$140$$
$$\tfrac{1}{3} = \$60$$
$$\therefore \text{ whole of his money} = \tfrac{3}{3} = \$180.$$

5. $\frac{1}{4}$ of $\frac{24}{7} = \frac{1}{12}$; $\frac{3}{4}$ of remaining $\frac{11}{12}$, or $\frac{11}{16}$ is in the water. Hence in mud and water there is $\frac{1}{12}+\frac{11}{16} = \frac{25}{36}$; and in air, $1 - \frac{25}{36} = \frac{11}{36}$, which $= 5\frac{1}{2}$ ft.

$\therefore$ whole post, or $\frac{36}{36} = \frac{36\times11}{11\times2} = 18$ ft.

VI.—Page 73.

1. Art. 73.

2. Denominator must evidently be equal to sum of numerators. Hence fractions are $\frac{3}{15}$, $\frac{5}{15}$, and $\frac{7}{15}$.

3. $$2\tfrac{2}{5}\times(3\tfrac{3}{4}+4\tfrac{5}{6}-6\tfrac{6}{7})\div4\tfrac{1}{7}$$
$$= \tfrac{12}{5}\times\tfrac{145}{84}\times\tfrac{7}{29}$$
$$= 1.$$

4. $\frac{3}{4}$ cost of watch to B = \$36
$\therefore$ " " = $\frac{1}{3}$ of \$36
= \$48.

Again, $1\frac{1}{5}$ cost of watch to A = \$48
$\therefore$ " " = $\frac{5}{6}$ of \$48
= \$40.

5. Length of rooms $\frac{780}{36}$, $\frac{675}{36}$, $\frac{640}{36}$; and H. C. F. of 780, 675, and 640, is 5;

$\therefore \frac{5}{36}$ ft., or $1\frac{2}{3}$ in., is the longest ruler.

VII.—Page 73.

1. To obtain the product of the multiplier and multiplicand we perform the same operation on the multiplicand as we did on unity to obtain the multiplier.

Thus, to multiply $\frac{2}{3}$ by $\frac{3}{4}$, what was done with 1 to make $\frac{1}{4}$, the same must be done with $\frac{2}{3}$. But, to make $\frac{3}{4}$, 1 is divided into 4 equal parts, and three of them are taken. Hence, to make $\frac{2}{3}$ multiplied by $\frac{3}{4}$, $\frac{2}{3}$ must be divided into 4 equal parts, and 3 of them must be taken.

2. Wife and son had $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$; daughter had, therefore, $1-\frac{5}{6}=\frac{1}{6}$. Mother leaves $\frac{3}{5}$ of $\frac{1}{3}=\frac{1}{5}$ to son, and rest, $\frac{1}{3}-\frac{1}{5}=\frac{2}{15}$ to daughter; daughter then had $\frac{1}{6}+\frac{2}{15}=\frac{3}{10}$. Son's and daughter's shares make the whole, and of this daughter gets $\frac{1}{3}$. Hence daughter's gain $=\frac{1}{3}-\frac{3}{10}=\frac{1}{30}$.

3. No. that can read $=\frac{1}{2}$

" " & write $=\frac{21}{50}$ of $\frac{1}{2}=\frac{21}{100}$

" " " & cipher $=\frac{4}{25}\left\{1-\left(\frac{1}{2}+\frac{21}{100}\right)\right\}$

$=\frac{29}{625}$.

The rest $=1-\left(\frac{1}{2}+\frac{21}{100}+\frac{29}{625}\right)=\frac{609}{2500}=243600$;

$\therefore$ the whole population $=\frac{2500}{609}\times 243600$

$=1000000$.

4. The L. C. M. of 5, 6, $7\frac{1}{2}$, or of $\frac{10}{2}$, $\frac{12}{2}$, $\frac{15}{2}=\frac{60}{2}$

$=30$;

$\therefore$ they will all be together in 30 min.

A will go round it $\frac{30}{5}=6$ times;

B " " $\frac{30}{6}=5$ "

C " " $\frac{30}{7\frac{1}{2}}=4$ "

5. A now owns $\frac{5}{9} - \frac{3}{4}$ of $\frac{5}{9} = \frac{5}{9}(1 - \frac{3}{4}) = \frac{5}{36}$.
   B " $\frac{5}{12} - \frac{4}{7}$ of $\frac{5}{12} = \frac{5}{12}(1 - \frac{4}{7}) = \frac{5}{28}$.
   C " $\frac{5}{21} - \frac{9}{11}$ of $\frac{5}{21} = \frac{5}{21}(1 - \frac{9}{11}) = \frac{10}{231}$.
   D " $\frac{9}{11}$ of $\frac{5}{21}$ $= \frac{15}{77}$.

VIII.—Page 73.

1. Operations are more easily performed. Art. 64.

2. $3\frac{1}{2} - 2\frac{26}{21} + \frac{11}{84}$ of $2\frac{1}{3} - 1\frac{1}{7} = \frac{23}{42} + \frac{35}{39} - 1\frac{1}{7}$
$= \frac{35}{39} - \frac{25}{42}$
$= \frac{5}{3}(\frac{7}{13} - \frac{5}{14})$
$= \frac{55}{182}$.

And, $\frac{4}{21}$, $\frac{1}{6}$, $\frac{55}{182} = \frac{104}{546}$, $\frac{91}{546}$, $\frac{165}{546}$.

$\frac{104}{546} - \frac{91}{546} = \frac{13}{546}$; and $\frac{165}{546} - \frac{104}{546} = \frac{61}{546}$,

$\therefore \frac{104}{546}$ or $\frac{4}{21}$ is more nearly equal to $\frac{91}{546}$ or $\frac{1}{6}$ than to $\frac{165}{546}$.

Also, $\frac{61}{546} - \frac{13}{546} = \frac{48}{546} = \frac{8}{91}$.

3. Let 1 denote number of sovereigns; then

$1 - (\frac{1}{6} + \frac{2}{9} + \frac{1}{12} + \frac{2}{9} + \frac{3}{18}) = 5$,

or $\frac{5}{36} = 5$;

$\therefore 1 = 36$.

Hence whole number of sovereigns $= 36$.

4. Let A, B, C represent the horses.

A would go round the island in 2 min.

B " " " $2\frac{1}{2}$ "

C " " " 3 "

The L. C. M. of 2, $2\frac{1}{2}$, 3, is 30; hence in 30 minutes they will be together.

A goes round 15 times, and, therefore, travels

15×300 rods $= 4500$ rods.

B goes round 12 times, and, therefore, travels

12×300 rods $= 3600$ rods.

C goes round 10 times, and, therefore, travels

10×300 rods $= 3000$ rods.

Decimal Fractions.

I.—Page 99.

1. Art. 101.

2. Art. 102.

3. By actual division, $\frac{10}{9\cdot009} = 1\cdot11\frac{1}{9\cdot009}$; and

" " $\frac{10}{1\cdot11} = 9\cdot009\frac{1}{1\cdot11}$;

and since $\frac{1}{9\cdot009}$ is less than $\frac{1}{1\cdot11}$, evidently the first statement is more nearly correct.

4. Since division is only a short method of performing subtraction, divide 2·291 by ·0087, and quotient = 263, with remainder ·0029; which is $\frac{1}{3}$ of ·0087.

5. Art. 109.

$\frac{355}{113} = 3\cdot1415929$; hence limits of error lies between ·0000006 and ·0000009.

II.

1. Art. 110.

$\frac{5}{32} = \frac{5}{2^5}$; $\frac{77}{1100} = \frac{7}{100}$; $\frac{1820}{2912} = \frac{5}{2^3}$; $\frac{91}{560} = \frac{13}{80}$

$= \frac{13}{2^3 \times 10}$;

the preceding fractions can evidently be reduced to finite decimals.

$\frac{231}{385} = \frac{77}{96} = \frac{77}{2^5 \times 3}$; $\frac{79}{405} = \frac{79}{3^4 \times 5}$; the preceding fractions cannot be reduced to finite decimals.

2. Value = ·0625 × 16 × 200 × ·0093125 × $8

= $14.90.

3. $3\cdot714535 = 3\cdot714\frac{1}{2} + \ldots$ which is evidently more nearly equal to 3·715 than to 3·714.

4. $\frac{14.4+1.44}{14.4-1.44} = \frac{15\cdot84}{12\cdot96} = \frac{11}{9}$.

5. $\frac{5}{7} = \cdot7142$.

III.—Page 100.

1. The *advantages* are (1) that the addition, subtraction, multiplication and division of decimals can be performed by processes the same as in ordinary whole numbers, with only additional rules for placing the decimal points in the results; and (2) decimals can be compared with the same ease as in whole numbers, whereas vulgar fractions have to be reduced to a common denominator.

The *disadvantages* are in recurring decimals, which are only approximations.

2. $\cdot 475 = \frac{475}{1000} = \frac{19}{40}$; and $\cdot 3\dot{8} = \frac{7}{18}$.
Share of third $= \left\{1 - \left(\frac{7}{18} + \frac{19}{40}\right)\right\}$ of $\$6000 = \$816.66\frac{2}{3}$

3. Owns $\left(\frac{3}{5} - \frac{7}{9} \text{ of } \frac{3}{5}\right) = \frac{2}{15} = \cdot 1\dot{3}$;
and $\frac{7}{9}$ of $\frac{3}{5}$ of value $= \$1400$;
$\therefore$ whole value $= \frac{15}{7} \times \$1400 = \$3000$.

4. Horse $= \$120$
Buggy $= \$\frac{120}{3} + \$36\frac{5}{16} = \$76{\cdot}3125$
Harness $= \frac{185}{999}$ of $(\$120 + \$76{\cdot}3125)$
$= \$\frac{185 \times 196{\cdot}3125}{999} = \$36{\cdot}35416\dot{6}$
$\therefore$ entire outlay $= \$120 + \$76{\cdot}3125 + \$36{\cdot}35416\dot{6}$
$= \$232{\cdot}66\frac{2}{3}$.

NOTE.—*For method of dividing by 999 see Art. 51.*

5. $\cdot 6\dot{3} \times \cdot 136\dot{3} \times$ third fraction $= \frac{4}{7}$;
or $\frac{63}{99} \times \frac{123}{900} \times$ third fraction $= \frac{4}{7}$;
$\therefore$ third fraction $= \frac{11}{7} \times \frac{300}{41} \times \frac{4}{7} = \frac{13200}{2009}$.

IV.

1. *Decimals* are *fractions* having the denominator, which must be 10 or some power of 10, suppressed;

vulgar fractions may have *any number whatever* for denominator.

2. He gains 14 cents a yard on 140 yds. = \$19.60
He next gains $\frac{140}{35}$ yds., which at 50 cents a yard = 2.00

Net gain = \$21.60

3. If $\frac{18}{25} = 423$
$1 = \frac{25}{18} \times 423$
$\therefore \frac{34}{47} = \frac{34}{47} \times \frac{25}{18} \times 423 = 425.$

5. Correct length $= (84 - 7 \times 12 \times \cdot 0208\frac{1}{3})$ yds.
$= 82\frac{1}{4}$ yds.

V.—Page 101.

1. Arts. 109 and 110.

2. Fraction $= \frac{25 \times 39.371}{12 \times 5280} = \frac{1000 \times 39.371}{40 \times 12 \times 5280} = \frac{39371}{2534400}$.

3. 2700 mi. in 230 hrs. $= 11\frac{17}{23}$ miles an hour;
and 405 " 18 " $= 22\frac{1}{2}$ " "
then $22\frac{1}{2} - 11\frac{17}{23} = 10\frac{35}{46} = 10{\cdot}7608685$ &c.

4. div. + quot. $= 7\frac{1}{2}$
div. $= \frac{3}{7}$ quot.
rem. $= \frac{20}{27}$ div., and
$\therefore = \frac{20}{27}$ of $\frac{3}{7}$ quot. $= \frac{20}{63}$ quot.
$\therefore \frac{3}{7}$ quot. + quot. $= 7\frac{1}{2}$, or $\frac{10}{7}$ quot. $= 7\frac{1}{2}$;
$\therefore$ quot. $= 5\frac{1}{4}$.
But dividend = quot. × div. + rem.
$= \frac{21}{4} \times \frac{3}{7}$ of $\frac{21}{4} + \frac{20}{63}$ of $\frac{21}{4}$
$= 13\frac{23}{43}$.

5. B's $= A$'s $-$ \$46.70,
C's $= B$'s $-$ \$34.59 $= (A$'s $-$ \$46.70$) -$ \$34.59
$= A$'s $-$ \$81.29;

Sum of all the shares $= A$'s + B's + C's

$= A$'s $+ A$'s $-$ \$46·70 $+ A$'s $-$ \$81·29

$= 3A$'s $-$ \$127·99.

$\therefore 3A$'s $-$ \$127·99 $=$ \$448·715

$3A$'s $=$ \$576·705

A's $=$ \$192.23$\frac{1}{2}$.

B's $=$ \$192.23$\frac{1}{2}$ $-$ \$46·70 $=$ \$145·53$\frac{1}{2}$;

C's $=$ \$192·23$\frac{1}{2}$ $-$ \$81·29 $=$ \$110·94$\frac{1}{2}$.

VI.—Page 101.

1. Arts. 114 and 116.

2. Art. 109.

3. Sum left after the first spending $= \frac{18}{99}$ of money $-$ \$2$\frac{1}{2}$.

$= \frac{1}{5}$ of money $-$ \$2$\frac{1}{2}$.

As he spent $\frac{960}{1441}$ of ($\frac{1}{5}$ of money $-$ \$2$\frac{1}{2}$) $-$ \$1$\frac{15}{99}$, he had remaining $\frac{481}{1441}$ of ($\frac{1}{5}$ of money $-$ \$2$\frac{1}{2}$) $+$ \$1$\frac{15}{99}$, or

$= \frac{481}{1441 \times 5}$ of money $-$ \$$\frac{2405}{2882}$ $+$ \$1$\frac{15}{99}$,

$\therefore \frac{481}{1441 \times 5}$ of money $-$ \$$\frac{2405}{2882}$ $+$ \$1$\frac{15}{99}$ $=$ \$$\frac{2603}{990}$;

$\therefore \frac{481}{1441 \times 5}$ of money $=$ \$$(\frac{2603}{990} + \frac{2405}{2882} - 1\frac{15}{99})$

$=$ \$$\frac{594516}{9 \times 110 \times 262}$;

$\therefore$ the money $=$ \$$\frac{1441 \times 5 \times 594516}{481 \times 9 \times 110 \times 262}$

$=$ \$34$\frac{1}{3}$.

This example illustrates the utility of merely indicating the multiplication and division until the final result is required.

4. $\frac{1}{5} = \frac{2}{10} = \cdot 2$

$\frac{1}{5} \cdot \frac{1}{5^5} = \frac{1}{5^6} = \frac{2^6}{10^6} = \cdot 000064$;

$\therefore \frac{1}{5} + \frac{1}{5} \cdot \frac{1}{5^5} = \cdot 000064.$

Also,

$$\tfrac{1}{3}\cdot\frac{1}{5^3}=\tfrac{1}{3}\cdot\frac{2^3}{10^3}=\frac{2\cdot66}{10^3}=\cdot0026666$$

$$\tfrac{1}{7}\cdot\frac{1}{5^7}=\tfrac{1}{7}\cdot\frac{2^7}{10^7}=\frac{18\cdot2}{10^7}=\cdot0000018\ ;$$

$$\therefore\ \tfrac{1}{3}\cdot\frac{1}{5^3}+\tfrac{1}{7}\cdot\frac{1}{5^7}=0026685.$$

Therefore

$$16\times\left\{\tfrac{1}{3}-\tfrac{1}{3}\cdot\frac{1}{5^3}+\tfrac{1}{5}\cdot\frac{1}{5^5}-\tfrac{1}{7}\cdot\frac{1}{5^7}+\&c\right\}-2\tfrac{4}{3\,5}$$

$$=16\times\left\{\cdot200064-\cdot0026685\right\}-\cdot016736$$

$$=3\cdot141592.$$

5. $$\frac{1}{10^3}\times\left\{1-\frac{3}{10^2}+\frac{3\times4}{1\times2}\times\frac{1}{10^4}+\frac{3\times4\times5}{1\times2\times3}\times\frac{1}{10^6}\right\}$$

$$=\frac{1}{10^3}\times\left\{1-\frac{3}{10^2}+\frac{6}{10^4}+\frac{10}{10^6}\right\}$$

$$=\frac{1}{10^3}\times\left\{\frac{10^5-3\times10^3+6\times10+1}{10^5}\right\}$$

$$=\frac{97061}{10^8}$$

$$=\cdot00097061.$$

EXAMINATION PAPERS.

I.—Page 146.

1. 3 min. 56 sec., or 236 sec. = difference for 1 day.
   1 sec. = " $\frac{1}{236}$ "
   24 hr. or $24\times60\times60$ sec. = diff. for $\frac{24\times60\times60}{236}$ days
   $=\frac{6\times60\times60}{59}=6\times60\times1\frac{1}{59}$
   $=366\frac{6}{59}$ days.

2. Time to pass over 91713000 mi. = 8 min. 18 sec.

Time to pass over 592200×91713000 mi.
$= 592200 \times (8 \text{ min. } 18 \text{ sec.})$
$= 59220 \times 83$ min.
$= 3413$d. 9hr. (between 9 and 10 years.)

3. In a period of 400 years there are 97 leap years; (Art. 151).

$\therefore$ 400 $\times$ (5 hr. 48 min. 49·7 sec.) should be 97 days.
But 400 $\times$ (5 hr. 48 min. 49·7 sec.) = 96 d. 21 hr. 31 min. 20 sec.;

$\therefore$ in 400 yr. the error = 2 hr. 28 min. 40 sec.;
$\therefore$ in 12000 yr. " = 30(2 hr. 28 min. 40 sec.)
= 3 d. 2 hrs. 20 min.

4. In 8505 days there are 1417 weeks and 3 d. over;
$\therefore$ the first number appeared on a Friday.

8505 working days $= \frac{8505 \times 7}{6}$ ordinary days = 27 yrs. 61 da. nearly.

27 yrs. and 61 days from Monday, June 18th, 1877, is Friday, April 19th, 1850.

5. The time between 9 hr. 13 min. A.M. on June 26, 1858, and midnight on Dec. 31, 1873, is 5667 d. 14 hr. 47 min. Now 29 d. 12 hrs. 47 min. 30 sec. is contained in 5667 d. 14 hr. 47 min. 191 times and 26 d. 19 hr. 34 min. 30 sec. over.

$\therefore$ there were 191 full moons, and the last one occurred 26 d. 19 hr. 34 min. 30 sec. before 12 P.M. of Dec. 31, or at 4 hr. 25 min. 30 sec. A.M. of Dec. 4.

II.—Page 146.

1. Since 1 ft. 6 in. $= \frac{1}{2}$ a yard;
$\therefore$ 9 mi. 7 fur. 39 per. 5 yd. 1 ft. 9 in. = 10 mi. 3 in. which can easily be changed to inches, and the resulting number of inches reduced to 10 mi. 3 in.

2. No. of revolutions of fore-wheel $= \frac{7 \times 5280}{11} = 3360$;
$\therefore$ " of hind-wheel $= 3360 - 718 = 2642$

Hence the circumfer. of hind-wheel $= \frac{7 \times 5280}{2642}$ ft.
$= 13\frac{1307}{1321}$ ft.

3. Time in seconds $= \frac{333 \times 5280}{66} = 26640$
$= 7$ hr. 24 min.;

$\therefore$ It will reach Montreal at $(6.25 + 7.24)$ or 1.49 p.m.
Time the Toronto train has been going at 8 = 1 hr. 35 min.
Distance it goes in 1 hr. 35 min. $= \frac{5700 \times 66}{5280}$ mi.
$= 71\frac{1}{4}$ mi.

Distance between Montreal and Toronto train at 8 a.m. is $(333 - 71\frac{1}{4})$ mi., or $261\frac{3}{4}$ mi.

Each second they approach $(88 + 66)$ ft. or 154 ft.

Number of seconds to meet $= \frac{261.75 \times 5280}{154}$
Distance gone by Montreal train $= \frac{261.75 \times 5280 \times 88}{5280 \times 154}$ mi.
$= 149\frac{4}{7}$ mi.

4. Average length $= \frac{16050 \times (202 \text{ yd. } 9 \text{ in.})}{93}$
$= 19$ mi. $1464\frac{27}{62}$ yd.

5. Number of strokes $= \frac{2 \times 26 \times 1760}{3\frac{1}{4}}$
$= 28160.$

III.—Page 147.

The corresponding unit of area is a square each of whose sides is equal to the lineal unit, and the corresponding unit of volume is a cube each of whose edges is equal to the lineal unit.

When the lineal unit is twelve inches, the unit of area is a square each of whose sides is 12 inches, or a square whose area is 144 sq. in.; the unit of volume is a cube each of whose edges is 12 inches, or a cube whose volume is 1728 inches.

2. Length of table $= 90$ in.
Width of table $= 40$ in.
Area of table $= (90 \times 40)$ sq. in.;
$\therefore$ number of coins $= 90 \times 40 = 3600$.
3600 half pence $=$ £7 10s.

3. If A gets 1, B gets 2, and C $\frac{3}{4}$ of 3, or $2\frac{1}{4}$;
$1+2+2\frac{1}{4} = 5\frac{1}{4}$;
$\therefore$ A gets $\frac{1}{5\frac{1}{4}}$ of 17 a. 2 r. 38 per. 19 yd. 7ft. 45 in.
$=$ 3 a. 1 r. 20 per. 21 yd. $77\frac{1}{7}$ in.
and B gets $2 \times$ (3 a. 1 r. 20 per. 21 yd. $77\frac{1}{7}$ in.)
$=$ 6 a. 3r. 1 per. 11 yd. 7 ft. $118\frac{2}{7}$ in.
and C gets $2\frac{1}{4} \times$ (3 a. 1 r. 20 per. 21 yd. $77\frac{1}{7}$ in.)
$=$ 7 a. 2 r. 16 per. 17 yd. 1 ft. $29\frac{4}{7}$ in.

4. Number of yards in 1 bale $= \frac{67048}{68}$,
" " 1 piece $= \frac{67048}{34 \times 68}$
$= 29$.

5. Number of sq. in. in 15 sq. ft. $= 15 \times 144$,
$\therefore$ pressure $= (15 \times 144 \times 15)$ lb.
$= 16$ t. 4 cwt.

When the barometer is at 29 the pressure will evidently be $\frac{1}{30}$ less than before.
$\frac{1}{30}$ of 16 t. 4 cwt. $=$ 10 cwt. 3 qrs. 5 lb.

IV.—Page 147.

1. 2 bu. 3 pk. 3 qt. $= 91$ qt.
$\therefore$ cost $= 91 \times 12\frac{1}{2}$ cts.
$= \$11.37\frac{1}{2}$.

2. 130 rods 4 yd. $2\frac{1}{2}$ ft. $= 130\frac{9}{55}$ rods.
$\therefore$ cost $= 130\frac{9}{55} \times \$2.50$.
$= \$327\frac{13}{66}$.
Part to be paid in wheat $= \$227\frac{13}{66}$
$\therefore$ Number of bushels $= \frac{227\frac{13}{66}}{87\frac{1}{2}}$
$= 259$ bu. 2 pk. 1 gal. $1\frac{13}{21}\frac{3}{1}$ pt.

3. 29 gal. 3 qt. 1 pt. $= 29\frac{1}{8}$ gal.

$\therefore$ cost of brandy $= 29\frac{1}{8} \times 43\frac{3}{4}$ cts.;

$\therefore$ quantity of rye $= \dfrac{29\frac{1}{8} \times 43\frac{3}{4}}{31\frac{1}{4}}$ bu.

$= 41$ bu. 3 pk. $2\frac{2}{5}$ qt.

4. 111 bu. 2 pk. 4 qt. $= 3572$ qt.

2 bu. 1 pk. 4 qt. $= 76$ qt.

$\therefore$ number of bags $= \frac{3572}{76} = 47.$

5. Number of quarts $= \dfrac{129 \times 95 \times 4\frac{1}{2}}{8}$

$\therefore$ value of produce $= \dfrac{129 \times 95 \times 4\frac{1}{2} \times 45}{32 \times 8}$ cts.

$= \$96.93....$

V.—Page 148.

1. Number of ounces bought $= 12 \times 16,$

" " sold $= \frac{12 \times 7000}{20 \times 24}.$

Cost price $= 12 \times 16 \times 37\frac{1}{2}$ cents

$= \$72.$

Selling price $= \frac{12 \times 7000 \times 40}{20 \times 24}$ cents

$= \$70;$

$\therefore$ he loses \$2.

2. Cost of 1 oz. or 480 grs. $= 15$ cents;

$\therefore$ cost of $\frac{7000}{16}$ grs. $= \frac{7000 \times 15}{16 \times 480}$ cents

$= 13\frac{43}{64}$ cents.

3. It is evident the weight must be a common measure of 8 lb. 20 gr. and 8 lb. 11 oz. 16 dwt. 16 gr.

8 lb. 20 gr. $= 56020$ gr.

8 lb. 11 oz. 16 dwt. 16 gr. $= 51760$ gr.

The H. C. F. of 56020 gr. and 51760 gr. is 20 gr.

4. 39 mi. 1 fur. 1 per. 9 inch. = 2479167 in.
$\therefore$ the weight = 24791·67 lb. (Art. 138).
= 12 t. 7 cwt. 3 qr. 16·67 lb.

5. The thirtieth part of 1 cwt. 3 lb. = $\frac{23}{672}$ cwt.
The eighty-fourth part of $2\frac{1}{2}$ cwt. = $\frac{20}{672}$ cwt.
$\therefore$ 500 times their difference = $500 \times \frac{3}{672}$ cwt.
= 250 lb.

Examples (lxxxvii). Page 151.

		£	s.	d.	£	s.	d.	
1.								
40	23 ×	10	0	0	= 230	0	0	= cost of 23 rd.
30¼	4 ×	0	5	0	= 1	0	0	= " 4 p.
	1½ ×	0	0	$1\frac{1}{21}$	= 0	0	$8\frac{1}{21}$	= " 4½ yd.
					231	0	$8\frac{1}{21}$	= entire cost.
2.		£	s.	d.	£	s.	d.	
4	12 ×	3	18	2	= 46	18	0	= cost of 12 cwt.
25	3 ×	0	19	$6\frac{1}{2}$	= 2	18	$7\frac{1}{2}$	= " 3 qr.
16	22 ×	0	0	$9\frac{9}{50}$	= 0	17	$2\frac{9}{25}$	= " 22 lb.
	12 ×	0	0	$\frac{469}{800}$	= 0	0	$7\frac{7}{100}$	= " 12 oz.
					50	14	$4\frac{7}{100}$	= entire cost.
3.		£	s.	d.	£	s.	d.	
4	10 ×	2	18	$10\frac{2}{3}$	= 29	8	$10\frac{2}{3}$	= cost of 10 a.
40	3 ×	0	14	$8\frac{2}{3}$	= 2	4	2	= " 3 ro.
	26 ×	0	0	$4\frac{5}{12}$	= 0	9	$6\frac{5}{6}$	= " 26 p.
					32	2	$7\frac{1}{2}$	= entire cost.
4.		£	s.	d.	£	s.	d.	
4	132 ×	3	14	$8\frac{1}{4}$	= 492	18	9	= cost of 132 cwt.
25	3 ×	0	18	$8\frac{1}{16}$	= 2	16	$0\frac{3}{16}$	= " 3 qr.
	$10\frac{1}{2}$ ×	0	0	$8\frac{77}{80}$	= 0	7	$10\frac{117}{160}$	= " $10\frac{1}{2}$ lb.
					496	2	$7\frac{47}{160}$	= entire cost.
5.		£	s.	d.	£	s.	d.	
4	63 ×	12	12	0	= 793	16	0	= cost of 63 cwt.
25	3 ×	3	3	0	= 9	9	0	= " 3 qr.
	$17\frac{1}{2}$ ×	0	2	$6\frac{6}{25}$	= 2	4	$1\frac{1}{5}$	= " $17\frac{1}{2}$ lb.
					805	9	$1\frac{1}{5}$	= entire cost.

	£	s.	d.	£	s.	d.	
6.							
4 29 ×	105	0	0	= 3045	0	0	= cost of 29 a.
40 3 ×	26	5	0	= 78	15	0	= " 3 ro.
5 ×	0	13	$1\frac{1}{2}$	= 3	5	$7\frac{1}{2}$	= " 5 per.
				3127	0	$7\frac{1}{2}$	= entire cost.
7.							
20 16 ×	3	17	6	= 62	0	0	= cost of 16 oz.
24 6 ×	0	3	$10\frac{1}{2}$	= 1	3	3	= " 6 dwt.
20 ×	0	0	$1\frac{5}{16}$	= 0	3	$2\frac{3}{4}$	= " 20 gr.
				63	6	$5\frac{3}{4}$	= entire cost.
8.							
4 25 ×	42	2	4	= 1052	18	4	= cost of 25 a.
40 1 ×	10	10	7	= 10	10	7	= " 1 ro.
10 ×	0	5	$3\frac{7}{40}$	= 2	12	$7\frac{3}{4}$	= " 10 p.
				1066	1	$6\frac{3}{4}$	= entire cost.
9.							
4 13 ×	22	8	0	= 291	4	0	= cost of 13 cwt.
25 3 ×	5	12	0	= 16	16	0	= " 3 qr.
17 ×	0	4	$5\frac{19}{25}$	= 3	16	$1\frac{3}{25}$	= " 17 lb.
				311	16	$1\frac{3}{25}$	= entire cost.
10.							
4 319 ×	2	12	6	= 837	7	6	= cost of 319 cwt.
25 3 ×	0	13	$1\frac{1}{2}$	= 1	19	$4\frac{1}{2}$	= " 3 qr.
16 ×	0	0	$6\frac{3}{10}$	= 0	8	$4\frac{8}{10}$	= " 16 lb.
				839	15	$3\frac{3}{10}$	= entire cost.

Examples (xc.) Page 157.

1. Value of $\frac{3}{5}$ of estate = \$7520;

$\therefore$ " $\frac{5}{8}$ " $= \$\left(\frac{5}{8} \times \frac{7520}{\frac{3}{8}}\right)$

$= \$7833\frac{1}{3}$.

2. Value of $\frac{2}{3}$ of $\frac{3}{8}$ of ship $= \$1260$;
$\therefore$ " whole ship $= \$(4 \times 1260)$
$= \$5040.$

3. Quantity bought for 366 half-pence $= 3\frac{2}{5}$ lb.
$\therefore$ " " 2013 half-pence $= \frac{2013 \times 3\frac{2}{5}}{366}$ lb.
$= 18\frac{7}{10}$ lb.

4. Amount of work done in 25 da. $= \frac{2}{11}$;
$\therefore$ " " $11\frac{2}{3}$ $= \frac{11\frac{2}{3} \times \frac{2}{11}}{25}$
$= \frac{14}{165}$.

5. Time he walks 96800 ft. $= 330$ min.;
$\therefore$ " " 7920 ft. $= \frac{7920 \times 330}{96800}$ min.
$= 27$ min.

6. Value of $\frac{2}{7}$ of $\frac{5}{16}$ of $\frac{3}{14}$ of estate $= \$603.125$;
$\therefore$ " $\frac{1}{6}$ of $\frac{3}{16}$ of estate $= \$\frac{\frac{1}{6} \text{ of } \frac{3}{16} \times 603{\cdot}125}{\frac{2}{7} \text{ of } \frac{5}{16} \text{ of } \frac{3}{14}}$
$= \$1182.125.$

7. Distance 15.5 cwt. is carried $= 60$ mi.
$\therefore$ " 3.25 cwt. " $= \frac{15.5 \times 60}{3.25}$
$= 286\frac{2}{13}$ mi.

8. Value of $\frac{1}{9}$ of $\frac{7}{8}$ of $\frac{3}{11}$ of vessel $= \$1400$;
$\therefore$ " $\frac{1}{11}$ of $\frac{1}{2}$ of vessel $= \$\frac{\frac{1}{11} \text{ of } \frac{1}{2} \times 1400}{\frac{1}{9} \text{ of } \frac{7}{8} \text{ of } \frac{3}{11}}$
$= \$400.$

9. Value of 1 lb. of gold $= 12 \times £3{\cdot}89$;
$\therefore$ " ·04 " $= £(\cdot 04 \times 12 \times 3{\cdot}89)$
$= £1\ 17s.\ 4{\cdot}128d.$

10. Cost of 6 in. of the first kind $= \frac{9}{8\frac{1}{2}}d. = \frac{18}{17}d.$
" " second " $= \frac{11}{10\frac{1}{2}}d. = \frac{44}{41}d.$
Now $\frac{18}{17} = \frac{738}{17 \times 41}$, and $\frac{44}{41} = \frac{748}{17 \times 41}$;
$\therefore$ the first kind is the cheaper.

11. Weight carried 20 mi. for £$14\frac{1}{2}$ = 60 cwt. ;

$\therefore$ " " £$5\frac{7}{16}$ = $\frac{5\frac{7}{16}\times 60}{14\frac{1}{2}}$ cwt.

= $22\frac{1}{2}$ cwt.

Examples (xci). Page 158.

1. Amount mowed by 8 men in 7 da. = 40 a. ;

$\therefore$ " " 24 " 28 " = $\frac{24\times 28\times 40}{8\times 7}$ a.

= 480a.

2. Sum earned by 8 men in 5 da. = $60 ;

$\therefore$ " " 32 " 24 " = $\$\frac{32\times 24\times 60}{8\times 5}$

= $1152.

3. Number needed to consume 351 qr. in 168 da.

= 939 men ;

$\therefore$ " " " 1404 qr. in 56 da.

= $\frac{1404\times 168\times 939}{351\times 56}$ men

= 11268.

4. Number of horses supplied by 8 bu. for 16 da. = 2 ;

$\therefore$ " " " 3000 × 8 bu. for 24 da.

= $\frac{3000\times 16\times 2}{24}$

= 4000.

5. Cost of carriage of 3 cwt., 150 mi. = $12 ;

$\therefore$ " " 7·89 cwt., 50 mi. = $\$\frac{7.89\times 50\times 12}{3\times 150}$

= $10.52.

6. Cost of carriage of 2 t. for 6 mi. = $1.50 ;

$\therefore$ " " $12\frac{17}{20}$ t. for 34 mi. = $\$\frac{12\frac{17}{20}\times 34\times 1.50}{2\times 6}$

= $54·61¼.

7. Sum earned by 3 men in 4 da. = $15 ;

$\therefore$ " " 18 " 16 da. = $\$\frac{18\times 16\times 15}{3\times 4}$

= $360.

8. Number required by 6 people for 24 da. = 4 bu.
∴ " " 72 " 8 da.
$= \frac{72 \times 8 \times 4}{6 \times 24}$ bu.
= 16 bu.

9. Time required to travel 150 mi. = 60 hr.;
∴ " " " 500 mi. $= \frac{500 \times 60}{150}$ hr.
= 200 hr.
But 200 hr. $= \frac{200}{10}$ da. = 20 da.

10. Cost of carriage of $5\frac{1}{2}\frac{7}{8}$ cwt. = \$15.70;
∴ " " $4 \times 7\frac{1}{3}\frac{1}{6}$ cwt. $= \$\frac{4 \times 7\frac{1}{3}\frac{1}{6} \times 15.70}{5\frac{1}{2}\frac{7}{8}}$
= \$78.60.

11. Number required to earn \$120 in 6 da. = 16;
∴ " " " \$270 in 8 da.
$= \frac{270 \times 6 \times 16}{120 \times 8}$
= 27.

12. Number supplied for \$1.20 for 50 hr. = 5;
∴ " " \$21.60 for 60 hr.
$= \frac{21.60 \times 50 \times 5}{1.20 \times 60}$
= 75.

13. Time \$190 lasts 3 men = 4 weeks;
∴ " \$475 " 5 " $= \frac{475 \times 3 \times 4}{190 \times 5}$ weeks
= 6 weeks.

14. Cost of 2 horses for 5 mos. = \$120;
∴ " 3 " 11 " $= \$\frac{3 \times 11 \times 120}{2 \times 5}$
= \$396.

15. Time 5 horses are fed for 7050*d.* = 6 weeks;
∴ " 3 " " 4935*d.* $= \frac{5 \times 4935 \times 6}{3 \times 7050}$ w.
= 7 weeks.

16. Time required by 5 men to reap $12\frac{1}{2}$ a. $= 56$ hr.;
$\therefore$ " " 7 " 15 a.
$= \frac{5 \times 15 \times 56}{7 \times 12\frac{1}{2}}$ hr.
$= 48$ hr. $= 4$ da.

17. Quantity required for 858 men for 6 mo. $= 234$ qr.;
$\therefore$ " " 979 " $3\frac{1}{2}$ mo.
$= \frac{979 \times 3\frac{1}{2} \times 234}{858 \times 6}$ qr.
$= 155\frac{3}{4}$ qr.

18. Time in which 5 men earn \$315 $=$ 6 weeks;
$\therefore$ " " 4 " \$231 $= \frac{5 \times 231 \times 6}{4 \times 315}$ w.
$= 5\frac{1}{2}$ weeks.

19. Time in which 7 men mow 22 a. $= 88$ hr.;
$\therefore$ " " 12 " 360 a.
$= \frac{7 \times 360 \times 88}{12 \times 22}$ hr.
$= 840$ hr. $= 84$ da.

20. Time in which 10 horses eat $7\frac{1}{2}$ bu. $= 7$ da.;
$\therefore$ " " 28 " 30 bu. $=$
$= \frac{10 \times 30 \times 7}{28 \times 7\frac{1}{2}}$ da.
$= 10$ da.

21. Time 300 bbl. supply $(3 \times 44 \times 30)$ rounds $= 5$ da.;
$\therefore$ " 400 " $(5 \times 66 \times 40)$ "
$= \frac{400 \times 3 \times 44 \times 30 \times 5}{300 \times 5 \times 66 \times 40}$ da.
$= 2$ days.

22. Number required to earn 19314d. in 54 da. $= 29$;
$\therefore$ " " " $407 \times 20 \times 12d$. in 12 da.
$= \frac{407 \times 20 \times 12 \times 54 \times 29}{19314 \times 12}$
$= 660$.

23. Cost of hiring 3 horses for 1 mo. $=$ £18;
$\therefore$ " " 4 " 5 mo. $= £\frac{4 \times 5 \times 18}{3}$
$=$ £120.

Examples (xcii). Page 161.

1. Part of work done by A in 1 hr. $= \frac{1}{6}$.
" " B " $= \frac{1}{9}$;
$\therefore$ " " A and B " $= \frac{1}{6} + \frac{1}{9}$
$= \frac{5}{18}$.
Time required to do $\frac{5}{18}$ of work $= 1$ hr.;
$\therefore$ " " all the work $= \frac{18 \times 1}{5}$ hr.
$= 3\frac{3}{5}$. hr.

2. In 1 hr. A does $\frac{1}{35}$ of work; B, $\frac{1}{40}$; C, $\frac{1}{45}$.
$\therefore$ Part done by A, B and C in 1 hr.
$= (\frac{1}{35} + \frac{1}{40} + \frac{1}{45})$ of work
$= \frac{191}{2520}$ of work.
Time required to do $\frac{191}{2520}$ of work $= 1$ hr.;
$\therefore$ " " all the work $= \frac{2520 \times 1}{191}$ hr.
$= 13\frac{37}{191}$ hr.

3. In 1 day A and B reap $\frac{1}{3}$ of field; A and C, $\frac{2}{7}$; B and C, $\frac{1}{4}$;
$\therefore$ twice (A's work + B's work + C's work) daily
$= \frac{1}{3} + \frac{2}{7} + \frac{1}{4} = \frac{73}{84}$;
$\therefore$ A, B and C do $\frac{73}{168}$ of the work daily.
Time required to do $\frac{73}{168}$ of work $= 1$ da.;
$\therefore$ " " all the work $= \frac{168 \times 1}{73}$ da.
$= 2\frac{22}{73}$ da.

4. Part filled in 1 min. $= (\frac{1}{6} + \frac{1}{8} + \frac{1}{12})$ of vessel
$= \frac{3}{8}$ of vessel.
Time required to fill $\frac{3}{8}$ of vessel $= 1$ min.
$\therefore$ " " the vessel $= \frac{8 \times 1}{3}$ min.
$= 2\frac{2}{3}$. min.

5. Part done by A in 1 da. $= \frac{1}{14}$ of $\frac{7}{10}$ of work;
$\therefore$ " " 2 da. $= 2 \times \frac{1}{14}$ of $\frac{7}{10}$ "
$= \frac{1}{10}$ of work.

Part done by B in 2 da. $= 1-(\frac{7}{10}+\frac{1}{10}) = \frac{1}{5}$;
$\therefore$ " " 1 da. $= \frac{1}{2}$ of $\frac{1}{5} = \frac{1}{10}$;
$\therefore$ B would do the whole work in 10 days.

6. In 1 hr. A does $\frac{1}{3}$ of the work; B and C do $\frac{3}{5}$; A and C, $\frac{3}{4}$.

Part done by C in 1 hr. $= \frac{3}{4} - \frac{1}{3} = \frac{5}{12}$;
$\therefore$ " " B " $= \frac{2}{3} - \frac{5}{12} = \frac{3}{12}$.
Time B requires to do $\frac{3}{12}$ of work $= 1$ hr.;
$\therefore$ " B " all the work $= \frac{12 \times 1}{3}$ hr.
$= 4$ hr.

7. Part done by A in 12 da. $= \frac{12}{27}$;
" " B " 5 da. $= \frac{5}{15}$;
$\therefore$ " " C " 4 da. $= 1-(\frac{12}{27}+\frac{5}{15})$
$= \frac{2}{9}$.
Time C requires to do $\frac{2}{9}$ of work $= 4$ da.;
$\therefore$ " C " all the work $= \frac{9 \times 4}{2}$
$= 18$ days.

8. Part filled in 10 min. $= \frac{10}{18}+\frac{10}{20}-\frac{10}{40}$
$= \frac{29}{36}$.

EXAMINATION PAPERS.

I.—Page 164.

1. Weight carried 36 mi. $= 1200$ lb.;
$\therefore$ " " 24 mi. $= \frac{36 \times 1200}{24}$
$= 1800$ lb.

2. Value of $\frac{4}{5}$ of ship $= \$13056$;
$\therefore$ " $\frac{5}{8}$ " $= \$\frac{\frac{5}{8} \times 13056}{\frac{4}{9}}$
$= \$18360$.

3. Value of $12 \times 3\frac{3}{4}$ oz. of silver $= \$54$;
$\therefore$ " 22 oz. " $= \$\frac{22 \times 54}{12 \times 3\frac{3}{4}}$
$= \$26.40$.

4. Expenses in 35 da. $= \$61.60$;
$\therefore$ " 365 da. $= \$\frac{365 \times 61.60}{35}$
$= \$642.40$;
$\therefore$ his total income $= \$1042.40$.

5. When the tax is $6d.$ the income $= £1$;
$\therefore$ " " $3690d.$ " $= £\frac{3690 \times 1}{6}$
$= £615.$

II.—Page 164.

1. Tax on $\$2720 = \$(2720 - 2640{\cdot}66) = \$79.34$;
$\therefore$ " $\$1 = \frac{7934}{2720}$ cents.
$= 2{\cdot}9\frac{23}{136}$ cents.

3. Since 5 horses $= 84$ sheep;
$\therefore$ 10 " $= 168$ sheep;
$\therefore$ 10 horses and 132 sheep $= (168 + 132)$ sheep
$= 300$ sheep.
And 15 horses and 148 " $= (252 + 148)$ sheep
$= 400$ sheep.
Cost of keeping 300 sheep $= \$202$;
$\therefore$ " " 400 " $= \$\frac{400 \times 202}{300}$
$= \$269\frac{1}{3}$.

3. Debt on which he loses 25 cts. $= \$1$;
$\therefore$ " " $\$602.10 = \$\frac{60210 \times 1}{25}$
$= \$2408.40.$

4. No. required for 1 work in 22 da. $= 15$ men;
$\therefore$ " " 4 works " $\frac{22}{5}$ da. $= 4 \times 5 \times 15$ men
$= 300$ men.

5. Time for 72 men to do 1 work $= 63$ days
$\therefore$ " 42 " 3 works $= \frac{3 \times 72 \times 63}{42}$ da.
$= 324$ da.

III.—Page 164.

1. A's wages for $12\frac{6}{7}$ da. = A's wages for $7\frac{1}{2}$ da. + B's wages for $7\frac{1}{2}$ da.;

$\therefore$ A's wages for $(12\frac{6}{7} - 7\frac{1}{2})$ da. = B's wages for $7\frac{1}{2}$ da.;

$\therefore$ A's wages for $12\frac{6}{7}$ da. = B's wages for $\frac{12\frac{6}{7} \times 7\frac{1}{2}}{5\frac{5}{14}}$ da.

= 18 days.

2. No. required for 1 work in 30 da. = 100 men;

$\therefore$ " " 3 works in $\frac{30}{4}$ da. = $3 \times 4 \times 100$ men

= 1200 men.

3. In working capacity 5 men = 7 women;

$\therefore$ " " 7 men = $\frac{7 \times 7}{5}$ women

= $9\frac{4}{5}$ women.

Time for 7 women to do 1 work = 37 da.;

$\therefore$ " $(9\frac{4}{5} + 5)$ women to do 2 works = $\frac{2 \times 7 \times 37}{14\frac{4}{5}}$ da.

= 35 da.

4. Part done by A and B in 1 day = $\frac{1}{20}$ of work,

" " by B alone " = $\frac{1}{50}$ of work;

$\therefore$ " " by A alone " = $(\frac{1}{20} - \frac{1}{50})$ of work

= $\frac{3}{100}$ of work

$\therefore$ Time required by A to do all the work is $33\frac{1}{3}$ da.

Amount done by A in 20 da. = $\frac{20 \times 3}{100} = \frac{60}{100}$ of work.

" " B in 20 da. = $\frac{20 \times 2}{100} = \frac{40}{100}$ of work;

$\therefore$ A does $(\frac{20}{100}$ or $\frac{1}{5})$ of work more than B.

5. Part of cistern emptied in one min. = $\frac{1}{15} - \frac{1}{20}$

= $\frac{1}{60}$;

$\therefore$ time required to empty cistern = 60 min.

IV.—Page 165.

1. A works 10 da.; B, 3 da.; C, 4 da..

Work done by A in 10 da. = $\frac{10}{15}$ of work;

" B in 3 da. = $\frac{3}{15}$ " ;

$\therefore$ the time C does $\left\{1-(\frac{1}{15}+\frac{3}{18})\right\}$ of work $= 4$ da.;

$\therefore$ the time C does the entire work $= 6\times 4$ da.

$= 24$ da.

2. Time for $(9+2\times 12+3\times 7)$ boys $= 250$ da.;

$\therefore$ " $(18+2\times 15+3\times 9)$ "

$= \frac{(9+2\times 12+3\times 7)\times 250}{18+2\times 15+3\times 9}$ da.

$= 180$ da.;

$\therefore$ to do double the work they would be 2×180 da.

$= 360$ da.

3. They approach each other at the rate of 10 miles per hour;

$\therefore$ they would meet in $\frac{100}{10}$ hr., or 10 hr.;

$\therefore$ A would have gone 10×6 mi., or 60 mi.

When the sum of the distance each walks equals 50 mi., or 150 mi., they will be 50 mi. apart.

This is the case after they have walked $\frac{50}{10}$ hr., or $\frac{150}{10}$ hr. $= 5$ hr., or 15 hr., respectively.

4. Between noon Monday and $10\frac{1}{4}$ a.m. Saturday, there are $118\frac{1}{4}$ hr.

Time lost in 24 hr. $= 3\frac{1}{6}$ min.;

$\therefore$ " $118\frac{1}{4}$ hr. $= \frac{118\frac{1}{4}\times 3\frac{1}{6}}{24}$

$= 15$ min. $36\frac{7}{48}$ sec.

As the watch is 10 min. too fast, it will be 5 min. $36\frac{7}{48}$ sec. too slow.

5. The watch goes 290 min. in 300 min. of exact time;

$\therefore$ 290 min. on the watch $= 300$ min.;

$\therefore$ 300 " " $= \frac{300\times 300}{290}$ min.

$= 5$ hr. $10\frac{10}{29}$ min.

In the second case, 310 min. on the watch = 300 min.;

$\therefore$ 300 " " .

$$= \frac{300 \times 300}{310} \text{ min}$$

$$= 4 \text{ hr. } 50\tfrac{10}{31} \text{ min.}$$

V.—Page 165.

1. Wages for 60 da. at \$2 $= 60 \times \$2.$

$= \$120;$

$\therefore$ total loss by idleness $= \$28.$

Sum lost by being idle 1 day $= \$(2 + 1.50);$

$\therefore$ number of days of idleness $= \frac{28}{3.50} = 8;$

$\therefore$ he worked $(60 - 8)$ days $= 52$ days.

2. In 1 hr. 1 man does $\frac{1}{230}$ of work; 1 woman, $\frac{2}{115}$; 1 boy, $\frac{1}{138}$;

$\therefore$ part done hourly by 1 man, 2 women and 3 boys

$$= \tfrac{1}{230} + \tfrac{4}{115} + \tfrac{1}{46} = \tfrac{23}{230}.$$

Time required to do $\frac{23}{230}$ of work $= 1$ hr.;

$\therefore$ " " all the work $= \frac{230 \times 1}{23}$ hr.

$= 10$ hr.

3. By the second part of the question we see that pipe A − pipe C, empties $\frac{1}{40}$ of the cistern in 1 hr.;

$\therefore$ pipe C − pipe A, would fill $\frac{1}{40}$ " "

We have, therefore,

part of cistern filled in 1 hr. by pipe A + pipe $B = \frac{1}{4}$

" " " C − " $A = \frac{1}{40}$

" " " B − " $C = \frac{1}{60}$;

$\therefore$ " " 2 × " $B = \frac{1}{4} + \frac{1}{40} + \frac{1}{60} = \frac{7}{24}$

" " " $B = \frac{7}{48}$.

" " " $A = \frac{1}{4} - \frac{7}{48} = \frac{5}{48}$

$\therefore$ pipe B fills the cistern alone in $\frac{48}{7}$ hr. $= 6\frac{6}{7}$ hrs.

" A " $\frac{48}{5}$ hr. $= 9\frac{3}{5}$ hrs.

4. Number of hours between 12 on Saturday night and noon on Tuesday is 60.

Number of hours between 12 on Saturday night and 4 p.m. on Thursday is 112.

3603 min. on clock = 3600 min. of true time;

$\therefore$ 6720 " $= \frac{6720 \times 3600}{3603}$ min. "

$= 111$ hr. $54\frac{486}{1201}$ min.

$= 3$ hr. $54\frac{486}{1201}$ min. p.m. Thursday.

$= 3$ hr. $54\frac{2}{5}$ min. nearly "

5. Amount which the work falls behind daily

$= (\frac{1}{6}+\frac{1}{7}+\frac{1}{8}-\frac{1}{8}-\frac{1}{16})$ of a day's work.;

$\therefore$ in 84 days it falls behind

$84 \times (\frac{1}{6}+\frac{1}{7}+\frac{1}{8}-\frac{1}{8}-\frac{1}{16})$ " "

$= 17{\cdot}6$ days work;

$\therefore$ part which 17 men must do more $= {\cdot}6$ of a day's work;

$\therefore$ " 1 man " $= \frac{.6}{17}$ " "

$= \frac{3}{85}$ " "

Examples (xciv.) Page 169.

7. Time for which interest is to be calculated = 135 days.

Interest $= \$\frac{5913 \times 135 \times 15}{365 \times 200} = \$\frac{81 \times 27 \times 15}{200}$

$= \$164{\cdot}025.$

8. Time $= 159$ days $= \frac{159}{365}$ yr.

Interest $= £(204\frac{2}{240}\frac{11}{0} \times \frac{159}{365} \times \frac{5}{100})$

$= £\frac{49171 \times 53}{80 \times 7300}$

$= £4\ 9s.\ 2\frac{7189}{7300}d.$

Examples xcv. Page 171.

1. Interest on \$326 for 15 yr. = \$220.05;

$\therefore$ " \$100 for 1 yr. $= \$\frac{100 \times 220.05}{326 \times 15}$

$= \$4\frac{1}{2}.$

2. Interest on \$700 for 1 yr. $= \$\frac{700\times 6}{100} = \$42$
The entire interest on \$700 $= \$(920.50-700)$
$= \$220.50.$
Time for which \$42 is interest $= 1$ yr.;
$\therefore$ " " \$220.50 " $= \frac{220.50\times 1}{42}$ yr.
$= 5\frac{1}{4}$ yr.

3. Interest on \$100 for 8 mo. at 9 % $= \$6$;
$\therefore$ Principal which amounts to \$1 $= \$\frac{100}{106}$;
$\therefore$ " " " \$1325 $= \$\frac{1325\times 100}{106}$
$= \$1250.$

4. Sum on which \$54.00 is interest $= \$100$;
$\therefore$ " " \$202.50 " $= \$\frac{202.50\times 100}{54}$
$= \$375.$

5. Interest $=$ 2 Principal $-$ Principal.
Here the interest of Principal for 1 yr. at 5% is $\$\frac{5\times \text{Prin.}}{100}$.
Time to produce $\$\frac{5\times \text{Prin.}}{100} = 1$ yr.;
$\therefore$ " Prin. $= \frac{100}{5}$ yr. $= 20$ yr.

6. Interest on Principal for $16\frac{2}{3}$ yrs. $= \frac{7}{8}$ of Principal;
$\therefore$ " \$1 for 1 yr. $= \$\frac{\frac{7}{8}\times \text{Principal}}{\text{Principal}\times 16\frac{2}{3}}$;
$\therefore$ " \$100 for 1 yr. $= \$\frac{100\times 7\times \text{Principal}}{8\times \text{Principal}\times 16\frac{2}{3}}$
$= \$5\frac{1}{4}.$

7. Sum on which \$70 is interest $= \$100$;
$\therefore$ principal of which \$1 is the amount $= \$\frac{100}{170}$;
$\therefore$ " " \$1275 " $= \$\frac{1275\times 100}{170}$
$= \$750$
Time for which \$52.50 is interest $= 1$ yr.;
$\therefore$ " " \$(1406.25$-$750) " $= \frac{656.25\times 1}{52.50}$ yr.
$= 12\frac{1}{2}$ yr.

8. The interest on \$400 for 3 mo. = the interest on \$100 for 12 mo.

The interest on \$100 for 12 mo., at a certain rate % = the interest on \$200 for 12 mo. at half that rate;

∴ the sum borrowed would pay the same interest as \$(500 + 200) would.

Interest on \$700 for 1 yr. = \$35;

∴ " \$100 for 1 yr. = $\$\frac{100\times35}{700}$

= \$5.

9. Time for which $£\frac{730}{3\times16}$ is interest = 365 days;

∴ " $£4\frac{1}{24}$ " = $\frac{4\frac{1}{24}\times365}{\frac{730}{48}}$ da.

= 97 da.

10. Interest on $£556\frac{7}{8}$ for 125 da. = $£9\frac{1}{16}$;

∴ " £100 for 365 da. = $£\frac{100\times365\times9\frac{1}{16}}{556\frac{7}{8}\times125}$

= £4·752.

= $4\frac{3}{4}$ per cent.

11. Interest on \$8000 for 1 da = \$2;

∴ " \$100 for 365 da = $\$\frac{100\times365\times2}{8000}$

= $\$9\frac{1}{8}$.

12. Cost of wheat at end of 6 mo = 5000 × \$1.25

= \$6250.

Sum realized = \$6000.

Amount of \$6000 for 6 mo = \$6300.

∴ his gain = \$(6300 − 6250) = \$50.

13. Interest on Principal for $6\frac{1}{4}$ yr. = $\frac{3}{8}$ of Principal.

∴ " \$1 for 1 yr. = $\$\frac{\frac{3}{8}\times\text{Principal}}{\text{Principal}\times6\frac{1}{4}}$

∴ " \$100 "

$$= \$\frac{100\times\frac{3}{8}\times\text{Principal}}{\text{Principal}\times6\frac{1}{4}} = \$6.$$

14. Interest on \$100 for $4\frac{1}{2}$ yr. at 5 % = \$22.50 ;
∴ the Principal which amounts to \$1 = $\$\frac{100}{122.50}$;
∴ " " " \$735 = $\$\frac{735\times100}{122.50}$
= \$600.

Time for which \$30 is interest on \$600 = 1 yr. ;
∴ " " \$540 " " = $\frac{540\times1}{30}$ yr.
= 18 yr.
(18−$4\frac{1}{2}$) yr. = $13\frac{1}{2}$ yr.

15. Time for which \$94·7625 is interest = 365 da. ;
∴ " " \$37·905 "
= $\frac{37.905\times365}{94.7625}$ da.
= 146 da.

146 days from May 13th is Oct. 6th.

Examples (xcvi) Page 173.

1. Principal on Interest Jan. 1, 1877 = \$1500.00
Interest to March 16, 1877 = 18.25

Amount = \$1518.25
First payment = 100.00

Remainder = \$1418.25
Interest from March 16, to June 13, 1877 = 20.75

Amount = \$1439.00
Second Payment = 400.00

Remainder = \$1039.00
Interest from June 13, to Sept. 1 = 13.66

Amount = \$1052.66
Third Payment = 200.00

Remainder =	$852.66
Interest from Sept. 1, to Jan. 1, 1878 =	17.09
Amount =	$869.75
2. Principal on Interest March 15, 1876 =	$3500.00
Interest to June 1, 1876 =	44.87
Amount =	$3544.87
First Payment =	800.00
Remainder =	$2744.87
Interest from June 1, to Sept. 1 =	41.51
Amount =	$2786.38
Second Payment =	100.00
Remainder =	$2686.38
Int. from Sept. 1, 1876, to Jan. 1, 1877 =	53.87
Amount =	$2740.25
Third Payment =	1560.00
Remainder =	$1180.25
Interest from Jan. 1, to March 1, 1877 =	11.44
Amount =	$1191.69
Fourth Payment =	300.00
Remainder =	$891.69
Interest from March 1, to May 16, 1877 =	11.14
Amount =	$902.83
3. Principal on Interest, Oct. 15, 1859 =	$1200.00
Interest to Oct. 15, 1860 =	72.00

Amount =	\$1272.00
First Payment =	1000.00
Remainder =	\$272.00
Int. from Oct. 15, 1860, to April 15, 1861 =	8.16
Amount =	\$280.16
Second Payment =	200.00
Remainder =	\$80.16
Int. from April 15, 1861, to Oct. 15, 1861 =	2.40
Amount =	\$82.56

Examples (xcviii). Page 177.

1. Amount of \$1 $= \$(1.03)^4 = \$1{\cdot}125509$;
$\therefore$ " \$1000 $= 1000 \times \$1{\cdot}125509$
$= \$1125.509$;
$\therefore$ interest $= \$125.509$.

2. Amount of \$1 $= \$(1.03)^6 = \$1{\cdot}19405$;
$\therefore$ " \$200 $= 200 \times \$1{\cdot}19405$;
$= \$238.81$.

3. Interest of \$1 for 4 yr. $= \$(1.06^4 - 1) = \${\cdot}26248$.
" " 3 " $= \$(1.06^3 - 1) = \${\cdot}19102$;
$\therefore$ " " $3\frac{1}{2}$ "
$= \$\left({\cdot}19102 + \frac{{\cdot}26248 - {\cdot}19102}{2}\right)$
$= \${\cdot}22675$;
$\therefore$ interest of \$675.75 for $3\frac{1}{2}$ yr. $= \$(675.75 \times {\cdot}22675)$
$= \$153.22$.

4. Amount of \$1000 for 4 payments
$= \$(1000 \times 1.03^4)$
$= \$1125{\cdot}508\ldots$
Amount of \$1000 at simple int. $= \$1120.00$;
$\therefore$ his gain $= \$5{\cdot}508\ldots$

5. Amount of £5000 half-yearly $= £(5000 \times 1{\cdot}02^4)$
$= £5412{\cdot}1608$
Amount of £5000 yearly $= £(5000 \times 1.04^2)$
$= £5408$;
$\therefore$ difference $= £4{\cdot}1608$
$= £4\ 3s.\ 2\frac{74}{125}d.$

6. Interest on \$40000 $= \$\{40000 \times (1.05^4 - 1)\}$
$= \$8620.25.$
Interest on \$80000 $= \$\{80000 \times (1.05^2 - 1)\}$
$= \$8200.00$;
$\therefore$ the difference $= \$420.25.$

7. Compound interest of \$248 $= \$\{248 \times (1{\cdot}035^3 - 1)\}$
$= \$26.96\ldots$
Simple interest of \$248 $= \$(248 \times 3 \times {\cdot}035)$
$= \$26.04$;
$\therefore$ the difference $=$ 92 cents.

8. Amount of \$1 for 3 yr. $= \$(1.04)^3 = \$1{\cdot}124864.$
" " 2 yr. $= \$(1.04)^2 = \$1{\cdot}0816.$

Interest during 3rd yr. $= \${\cdot}043264$;
$\therefore$ Amount of \$1 for $2\frac{1}{2}$ yr. $= \$(1{\cdot}0816 + \frac{{\cdot}043264}{2}$
$= \$1{\cdot}103232.$

Hence the sum of which \$1·103232 is amount = \$1;
$\therefore$ " " \$16989·7728 "
$= \$\frac{16989{\cdot}7728}{1.103232}$
$= \$15400.$

9. The sum of which $\$(1.05)^3$ is the amount = \$1
$\therefore$ " " \$27783 " "
$= \$\frac{27783}{1.157625}$
$= \$24000.$

Examples (xcix). Page 181.

11. The sum of which $\$(1.05)^3$ is the Present Worth $= \$1$;

$\therefore$ " " $\$6945.75$ " "

$$= \$\frac{6945.75}{1.05^3}$$

$$= \$6000.$$

12. The amount of $\$1 = \$(1{\cdot}01375)^5 = \$1{\cdot}070668\ldots$;

$\therefore$ the discount off a debt of $\$1{\cdot}070668 = \${\cdot}070668$;

$\therefore$ " " $\$245.25$

$$= \$\frac{245.25 \times .070668}{1.070668}$$

$$= \$16{\cdot}186\ldots$$

13. The interest on $\$19{\cdot}3125$ for 1 yr.

$$= \$(20\tfrac{267}{960} - 19\tfrac{5}{16})$$

$$= \$\tfrac{927}{960};$$

$\therefore$ " " $\$100$ for 1 yr. $= \$\frac{100 \times \frac{927}{960}}{19\frac{5}{16}}$

$$= \$5.$$

14. The bill is due on May 4.

Hence the time is 73 days.

The sum of which $\$1.02$ is the present worth $= \$1$;

$\therefore$ " " $\$1127.10$ " "

$$= \$\frac{1127.10 \times 1}{1.02}$$

$$= \$1105.$$

15. Interest on $\$250$ for 1 time $= \$25$;

$\therefore$ " $\$250$ for 2 times $= \$50$;

$\therefore$ discount off $\$300$ for 2 times $= \$50$;

$\therefore$ " $\$275$ " $= \$\frac{275 \times 50}{300}$

$$= \$45\tfrac{5}{6}.$$

Again, interest on $\$250$ for $\frac{1}{2}$ time $= \$12.50$;

$\therefore$ discount off $\$262.50$ " $= \$12.50$;

$\therefore$ " $\$275$ " $= \$\frac{275 \times 12.50}{262.50}$

$$= \$13\tfrac{2}{21}.$$

16. The amount of \$1 = \$1·0375.

Hence, if \$1 is the cash price, \$1·0375 should be the credit price.

Now, \$1·0375 = $\$1\frac{3}{80}$.

Hence, if the cash price = 80,

the credit price = 83.

The credit price = \$33.20 ;

$\therefore$ the cash price = $\frac{80}{83}$ of \$33.20 ;

= \$32.

17. Interest on \$98 for 1 time = \$30 ;

$\therefore$ " \$98 for $\frac{1}{2}$ time = \$15 ;

$\therefore$ the discount off \$113 " = \$15 ;

$\therefore$ " \$128 " = $\$\frac{128 \times 15}{113}$

= $\$16\frac{12}{113}$.

18. Sum on which \$·80 is int. for 8 mo. = \$20 ;

$\therefore$ " " \$20.80 " " = $\$\frac{20 \cdot 80 \times 20}{\cdot 80}$

= \$520.

Interest on \$20 for 8 mo. = \$·80 ;

$\therefore$ " \$100 for 12 mo. = $\$\frac{100 \times 12 \times \cdot 80}{20 \times 8}$

= \$6.

Examples (c.) Page 183.

1. Interest on \$950 = $\$(950 \times \frac{1}{4} \times \frac{7}{100})$

= \$16·625.

True discount off \$950 = $\$\frac{950 \times 1\frac{3}{4}}{101\frac{3}{4}}$

= \$16·339... ;

$\therefore$ the difference = \$·285...

2. The bill is due on Sept. 20.

Interest on \$722.70 for 40 days at $7\frac{1}{2}$ %

= $\$(722.70 \times \frac{40}{365} \times \frac{7\frac{1}{2}}{100})$

= \$5.94 ;

$\therefore$ he received \$(722.70 − 5·94) = \$716.76.

3. The bill is due on Nov. 12.

Interest on \$7850 for 146 days at 10 %

$$= \$(7850 \times \tfrac{146}{365} \times \tfrac{10}{100})$$
$$= \$314.$$

4. The note is due on Oct. 6.

The interest on \$100 for 95 days $= \$\frac{133}{73}$;

$\therefore$ note for which he receives $\$(100 - \frac{133}{73}) = \$100$;

$\therefore$ " " \$501.69 $= \$\frac{501.69 \times 100}{\frac{7167}{73}}$

$$= \$511.$$

5. Interest on \$5555 $= \$(5555 \times \frac{6}{100})$

$$= \$333.30.$$

True discount off \$5555 $= \$\left(\frac{5555 \times 6}{106}\right)$

$$= \$314.43\ldots$$

$\therefore$ the difference $= \$18.86$.

EXAMINATION PAPERS.

I.—Page 184

1. *Compound* interest on \$1 $= \$(1.04^3 - 1)$;

$\therefore$ " " \$25000 $= 25000 \times \$(1.04^3 - 1)$

$$= 25000 \times \$\cdot 124864$$
$$= \$3121.60.$$

2. Amount of \$1 at compound interest $= \$1 \cdot 124864$.

" \$1 at simple " $= \$1.12$;

$\therefore$ sum on which \$·004864 is difference $= \$1$;

$\therefore$ " " \$3.80 " $= \$\frac{3.80 \times 1}{.004864}$

$$= \$781.25.$$

3. Compound interest on \$100 for 2 yr.

$$= 100 \times \$(1.04^2 - 1)$$
$$= \$8.16;$$

$\therefore$ the simple interest on \$100 for 2 yr. $= \$8.16$;

$\therefore$ " " \$100 for 1 yr. $= \$\frac{8.16}{2}$

$$= \$4.08.$$

4. Compound interest for 3 yr. $= 1000 \times \$(1.03^3 - 1)$
$= \$92{\cdot}727.$

" " 2 yr. $= 1000 \times \$(1.03^2 - 1)$
$= \$60.90\ ;$

$\therefore$ " " 3 yr. $= \$31{\cdot}827\ ;$

$\therefore$ " " 195 da. $= \frac{195}{365}$ of $\$31{\cdot}827$
$= \$17.00\ ;$

$\therefore$ " " 2 yr. and 195 da. $= \$77.90.$

5. He adds \$20 to his capital for each of 4 years.
Amount of the 1st \$20 saved $= 20 \times \$(1.04)^4.$
" 2nd \$20 " $= 20 \times \$(1.04)^3.$
" 3rd \$20 " $= 20 \times \$(1.04)^2.$
" 4th \$20 " $= 20 \times \$(1.04)\ ;$

$\therefore$ his capital is increased by
$\$(20 + 20 \times 1.04 + 20 \times 1.04^2 + 20 \times 1.04^3 + 20 \times 1.04^4)$
$= \$(20 \times 5{\cdot}41632\ldots) = \$108{\cdot}326\ldots\ ;$
$\therefore$ his present capital $= \$(8000 + 108{\cdot}326\ldots)$
$= \$8108{\cdot}326\ldots$

II.—Page 184.

1. See articles 181, 182.

2. True discount $= \$\frac{400 \times 5}{105} = \$\frac{400}{21}$
$= \$19.04\frac{16}{21}.$

Interest on $\$\frac{400}{21} = \$(\frac{400}{21} \times \frac{5}{100})$
$= \${\cdot}95\frac{5}{21}.$

Interest on \$400 $= \$\frac{400 \times 5}{100}$
$= \$20.$

Now, $\$(20 - 19.04\frac{16}{21}) = \$\,95\frac{5}{21}.$

3. Discount off £120 $=$ £10 ;
$\therefore$ " £110 $=$ £$\frac{110 \times 10}{120}$
$=$ £9 3*s.* 4*d.*

4. Interest on $\$(10292-372)$ for $1\frac{1}{5}$ yr. $= \$372$;
$\therefore$ " $\$100$ for 1 yr.

$$= \$\frac{100\times 372}{9920\times 1\frac{1}{5}}$$

$$= \$3\tfrac{1}{8}.$$

5. Present value of a debt of $\$(1.05^2) = \$1$;
$\therefore$ " " " $\$110.25$

$$= \$\frac{110.25\times 1}{1.05^2}$$

$$= \$100.$$

III.—Page 185.

1. Amount of $5000 at end of 18 mo. $= \$5450$.
This was the sum he had to return.
Amount of $7500 for 1 yr. $= \$7950$.
This was the sum he realized;
$\therefore$ he gained $\$(7950-5450) = \$2500$.

2. Discount on $7 for 93 days at 6 % $= \$\cdot 10701$;
$\therefore$ cash selling price $= \$7-\$\cdot 10701 = \$6\cdot 89299$.
Profit per cwt. $= \$6\cdot 89299 - \$5.25 = \$1\cdot 64299$.
Hence, total profit $= 43\frac{3}{4} \times \$1\cdot 64299 = \$71.88\ldots$

3. Present worth $= \$\frac{1000\times 100}{112.50}$

$$= \$888.88\tfrac{8}{9}.$$

See Note I., Art. 181.

4. Interest to be received each half year $= \$250$.
Interest on $1 for 1 mo. $= \$\frac{1}{240}$;

$$\text{Sum}\times\$(1\tfrac{6}{240}+1\tfrac{5}{240}+1\tfrac{4}{240}+1\tfrac{3}{240}+1\tfrac{2}{240}+1\tfrac{1}{240}) = \$250;$$

$$\therefore \text{ sum}\times\$(6\tfrac{21}{240}) = \$250;$$

$$\therefore \text{ sum} = \$\frac{250}{6\frac{21}{240}}$$

$$= \$41\tfrac{33}{487}.$$

NOTE.—The advanced student may refer to Ex. 1, page 342.

5. Interest on £266⅔ for ¼ yr. $= £(266\frac{2}{3} \times \frac{1}{4} \times \frac{3}{200})$
$= £3.$

Discount off £83 for ¼ yr. $= £\frac{83 \times 3\frac{1}{3}}{103\frac{1}{3}}$
$= £3.$

IV.—Page 185.

1. Interest on \$6400 for 8 mo. $= \$213\frac{1}{3} =$ discount
$\therefore$ \$6400 − \$213⅓ = \$6186⅔ = sum he has to hire.
Amount of \$1 for 8 mo. $= \$1.03\frac{1}{3}$;
$\therefore$ " \$6186⅔ " $= 6186\frac{2}{3} \times \$1.03\frac{1}{3}$
$= \$6392.88\frac{8}{9}$;
$\therefore$ sum gained $= \$(6400 - 6392.88\frac{8}{9})$
$= \$7.11\frac{1}{9}.$

2. Compound interest on $= \$(1.08^3 - 1)$
$= \$.259712.$
Simple interest on \$1 $= \$.24$;
$\therefore$ difference $= \$.019712.$
Sum on which \$·019712 is difference $= \$1$;
$\therefore$ " " \$985·60 " $= \$\frac{985.60 \times 1}{.019712}$
$= \$50000.$

3. Since the discount is the present worth of the interest,
Interest on £63$\frac{17}{20}$ for 2 yr. $= £7\ 19s.\ 7\frac{1}{2}d.$;
$\therefore$ " £63$\frac{17}{20}$ for 1 yr. $= £3\frac{317}{320}$;
$\therefore$ " £100 for 1 yr. $= £\frac{100 \times 3\frac{317}{320}}{63\frac{17}{20}}$
$= £6\frac{1}{4}.$

Again, sum on which £7$\frac{157}{160}$ is interest $= £63\frac{17}{20}$;
$\therefore$ " " £71$\frac{133}{160}$ "
$= £\frac{71\frac{133}{160} \times 63\frac{17}{20}}{7\frac{157}{160}}$
$= £574\ 13s.$

4. Amount of \$8000 in 4 yr. $= 8000 \times \$(1.05)^4$
$= \$9724.05$;
$\therefore$ A's is better by $\$(9724.05 - 9500)$
$= \$224.05$.

5. Suppose he borrows \$100,
then the interest he receives amounts to
$2 \times \$(1 + 1.02 + 1.02^2 + 1.02^3) = \$8{\cdot}243216$.
Interest he has to pay $= \$6$;
$\therefore$ sum on which he gains $\$2{\cdot}243216 = \$100$;
$\therefore$ " " $\$269{\cdot}18592 = \$\frac{269{\cdot}18592 \times 100}{2{\cdot}243216}$
$= \$12000$.

Examples (ci.) Page 187.

5.

$0 \times \frac{1}{6} = 0$
$3 \times \frac{1}{6} = \frac{3}{6}$
$6 \times \frac{1}{6} = \frac{6}{6}$
$9 \times \frac{1}{6} = \frac{9}{6}$
$12 \times \frac{1}{6} = \frac{12}{6}$
$15 \times \frac{1}{6} = \frac{15}{6}$

$\frac{6}{6}$ $\frac{45}{6}$

$\therefore$ the equated time $= \frac{\frac{45}{6}}{\frac{6}{6}} = 7\frac{1}{2}$ mo.

6. $16 \times 450 = 7200$
$13\frac{1}{2} \times 250 = 3375$

$700 \;)\; 10575$

$15\frac{3}{28}$ = equated time.

It is now required to find the present worth of \$700 due in $15\frac{3}{28}$ mo.

Present worth of $\$700 = \$\frac{700 \times 100}{105\frac{1}{28}}$
$= \$666\frac{12}{29}\frac{24}{41}$.

7.

$$2 \times \tfrac{1}{6} = \tfrac{2}{6}$$
$$3 \times \tfrac{2}{6} = \tfrac{6}{6}$$
$$6 \times \tfrac{3}{6} = \tfrac{18}{6}$$
$$\tfrac{6}{6}\ )\ \tfrac{26}{6}$$
$$4\tfrac{1}{3}.$$

The whole debt is due in $4\frac{1}{3}$ mo.;

$\therefore$ if one-half of it is paid now, the other should no be paid till $2 \times 4\frac{1}{3}$ mo., or $8\frac{2}{3}$ mo.

8.

Debt.	When Due.	No. of days from Jan. 30.
$80.75	Jan. 30	0
150.00	Apr. 3	63
30.80	July 1	152
40.50	Aug. 10	192
60.30	Aug. 25	207

Taking Jan. 30 as the date from which to calculate the equated time, we have

$$0 \times 80{\cdot}75 = 000000$$
$$63 \times 150.00 = 9450.00$$
$$152 \times 30.80 = 4681.60$$
$$192 \times 40.50 = 7776.00$$
$$207 \times 60.30 = 12422.10$$
$$362.35\ )\ 34329.70$$
$$95 \text{ almost.}$$

95 da. from Jan. 30 is May 5.

Time between May 5 and June 2 = 28 da.

Interest on $362.35 for 28 da. at 6 %

$$= \$(362.35 \times \tfrac{28}{365} \times \tfrac{6}{100})$$
$$= \$1.66\ldots;$$

$\therefore$ $(362.35 + 1.66\ldots), or $364.01... will balance the account.

9. £140 is due in 50 da.
£120 " 74 da.
£380 " 106 da.

$\therefore$ equated time $= \frac{7000+8880+40280}{140+120+380}$ da.

$= 88$ da. (nearly).

88 days from the 1st of March is 28th of May.

10.

Debt.	When Due.	No. of days from 13th Jan.
24418*d*.	Feb. 8	26
34594*d*.	March 5	51
72946*d*.	March 18	64
181688*d*.	May 13	120
29658*d*.	May 23	135
29658.*d*	June 5	143

$26 \times 24418 = 634868$
$51 \times 34594 = 1764294$
$64 \times 72946 = 4668544$
$120 \times 181688 = 21802560$
$135 \times 29658 = 4003830$
$143 \times 29658 = 4241094$

$372962 \;)\; 37115190$

100, nearly.

Examples (cii.) Page 190.

1.

Dr. J. Hughes in account with S. Adams. Cr.

July 4	0 × 375.90 =	.0	Aug. 10	0 × 316.00 =	.0
Aug. 20	47 × 815.58 =	38332.26	Sept. 1	22 × 675.00 =	14850.00
Aug. 29	56 × 178.25 =	9982.00	Sep. 25	46 × 512.25 =	23563.50
Sept. 25	83 × 387.20 =	32137.60	Nov. 20	102 × 161.75 =	16498.50
Dec. 5	154 × 418.70 =	64479.80	Dec. 1	113 × 100.00 =	11300.00
	2175.63	)144931 66		1765	)66212
	67, early.			nearly 38.	

67 days from July 4 is September 9.
Due September 9 ..... $2175.63.

38 days from Aug. 10 is Sept 17.
Due September 17..... $1765.

If \$2175.63 gain a certain interest in 8 days (Art. 185) \$410.63 will gain the same interest in

$$\frac{2175.63 \times 8}{410.63} \text{ days} = 42 \text{ days}.$$

42 days before Sept. 17 is Aug. 6.

2. The items of the Dr. side fall due Oct. 12, Nov. 14, Jany. 17, and Dec. 31, respectively.

Dr.		A. B. Conron.		Cr.
Oct. 12	0 × 927.30 = 0		Oct. 10	0 × 500 = 0
Nov. 14	33 × 342.75 = 11310.75		Nov. 20	41 × 300 = 12300.00
Dec. 31	80 × 175.50 = 14040.00		Nov. 30	51 × 250 = 12750.00
Jan. 17	97 × 212.13 = 20576.61			
	1657.68) 45927.36			1650) 25050.00
	nearly 28.			nearly 24.
28 days from Oct. 12 is Nov. 9.			24 days from Oct. 10 is Nov. 3.	
Due Nov. 9 ..... \$1657.68.			Due Nov. 3..... \$1050	

If \$1050 gain a certain interest in 6 da. the balance \$607.68, will gain the same interest in

$$\frac{1050 \times 6}{607.68} \text{ days} = 10\tfrac{1}{4} \text{ days, nearly.}$$

Hence, the balance will be due on the 11th day from Nov. 9, or on Nov. 20.

3.

Dr.		J. Green in account with Adam Miller & Co.		Cr.
March 1,	184 × 720.75 = 132618.00		April 1,	0 × 700.00 = 000000.00
" 20,	203 × 815.30 = 165505.90		May 30,	243 × 569.80 = 138483.27
April 11,	224 × 587.80 = 131667.20		July 20,	110 × 500.00 = 55000.00
" 30,	243 × 300.00 = 72900.00		Sept 25,	177 × 100.00 = 17700.00
June 15,	289 × 625.25 = 180697.25		" 30,	963 × 750.20 = 272392.60
July 18,	323 × 560.00 = 180880.00		Oct. 30,	394 × 329.96 = 130004.24
Aug. 30,	364 × 681.90 = 249303.60		Nov. 20,	414 × 500.00 = 207000.00
Sept. 25,	389 × 365.30 = 142101.70			
	4659.30) 1255673.65			3450.05) 820510.11
	nearly 269 da.			nearly 238 da.
269 days from March 1 is Nov. 25.			238 days from April 1 is Nov. 25.	
Due Nov. 25, \$4659.30.			Due Nov. 25, \$3450.05.	

Since both sides of the account fall due on Nov. 25, the account should be settled on that day.

Examples cv. Page 193.

5. Brokerage on \$578 = \$26.01;

$\therefore$ " \$100 = $\$\frac{100 \times 26.01}{578}$

= \$4½.

6. Commission on \$100 invested = \$$2\frac{1}{3}$;
∴ " " \$$102\frac{1}{3}$ sent = \$$2\frac{1}{3}$;
∴ " " \$3377 " = \$$\frac{3377 \times 2\frac{1}{3}}{102\frac{1}{3}}$
= \$77.

7. Ready money payment of \$100 = \$97.50;
∴ " " \$7680 = \$$\frac{7680 \times 97.50}{100}$
= \$7488.

8. If he sell wheat to the value of \$100 his commission = \$2, and he has \$98 to invest in silk.
Commission on \$98 = \$$\frac{98 \times 4}{104}$ = \$$3\frac{10}{13}$;
∴ total commission = \$$5\frac{10}{13}$;
∴ sum invested when \$$5\frac{10}{13}$ is the com. = \$$94\frac{3}{13}$;
∴ " " \$600 " .
= \$$\frac{600 \times 94\frac{3}{13}}{5\frac{10}{13}}$
= \$9800.

9. Sum on which \$1.50 is brokerage = \$100;
∴ " " \$576 is brokerage = \$$\frac{576 \times 100}{1.50}$
= \$38400.

10. Brokerage on \$100 invested = \$·25;
∴ " " \$100.25 given = \$·25;
∴ " " \$20050 " = \$$\frac{20050 \times .25}{100.25}$
= \$50.
Sum invested = \$(20050 − 50) = \$20000.

Examples (cvi). Page 194

3. Premium on £100 at $2\frac{1}{3}$ % = £$2\frac{1}{3}$;
∴ sum for which goods worth £$97\frac{2}{3}$ are insured = £100;
∴ " " £$4384\frac{1}{80}$ "
= £$\frac{4384\frac{1}{80} \times 100}{97\frac{2}{3}}$
= £4488 15*s*.

4. Annual payment on 2 policies of \$100 each
$= \$(3.75 + 3.80)$
$= \$7.55$;
Total payment on 2 policies of \$5500 each
$= \$(55 \times 7.55)$
$= \$415.25.$

5. Total payment for cargo worth \$100
$= \$(1\frac{1}{2} + \frac{1}{6} + \frac{1}{8})$
$= \$1\frac{19}{24}$;
$\therefore$ " " \$26400
$= \$(264 \times 1\frac{19}{24})$
$= \$473.$

6. Sum on which \$2.20 is premium $= \$100$;
$\therefore$ " " \$80.85 " $= \$\frac{80.85 \times 100}{2.20}$
$= \$3675.$

But \$3675 is $\frac{75}{100}$ of value of 500 bbl. of flour;
$\therefore$ value of 1 bbl. $= \$\frac{\frac{100}{75} \times 3675}{500} = \$9.80.$

7. $\frac{5}{200}$ of risk $= \frac{3}{100}$ of $\frac{4}{5}$ of risk + \$10;
$\therefore$ $\frac{5}{200}$ of risk $= \frac{12}{500}$ of risk + \$10;
$\therefore$ $(\frac{5}{200} - \frac{12}{500})$ of risk $= \$10$;
$\therefore$ $\frac{25-24}{1000}$ of risk $= \$10$
$\therefore$ risk $= \$(1000 \times 10)$
$= \$10000.$

8. Sum on which $\$2\frac{3}{8}$ is premium $= \$100$;
$\therefore$ " " \$71.25 " $= \$\frac{71.25 \times 100}{2\frac{3}{8}}$
$= \$3000.$

But \$3000 is only $\frac{5}{8}$ of the value of the apples;
$\therefore$ value of the apples $= \frac{8}{5}$ of \$3000 $= \$4800.$

Examples (cvii). Page 195.

1. Tax paid on \$100000 $= \$1050$;
$\therefore$ " " \$5400 $= \$\frac{5400 \times 1050}{100000}$
$= \$56.70.$

2. Tax on \$8500 = \$144.50;

$\therefore$ " \$1 = $\frac{14450}{8500}$ cents

= 1·7 cents.

3. Tax on \$80000 = \$1400;

$\therefore$ " \$75000000 = \$$\frac{75000000 \times 1400}{80000}$

= \$1312500.

4. Of each \$100 collected, \$96 is spent in paying for the school-house;

$\therefore$ \$8400 requires a tax of \$$\frac{8400 \times 100}{96}$ = \$8750.

Tax paid by \$700000 = \$8750;

$\therefore$ " \$1 = $\frac{8750}{700000}$ cents

= $1\frac{1}{4}$ cents.

Examples (cviii.) Page 196.

4. Cost of 8400 lb. = \$630.

Specific duty = 8400 × $\frac{1}{2}$ cts. = \$42.

Ad valorem duty = \$$\frac{630 \times 25}{100}$ = \$157.50;

$\therefore$ total duty = \$199.50.

5. Value of cotton on which \$17.50 is duty = \$100;

$\therefore$ " " " \$1662.50 "

= \$$\frac{1662.50 \times 100}{17.50}$

= \$9500.

EXAMINATION PAPERS.

I.—Page 197.

1. Gain on 246 drams = 10 drams;

$\therefore$ " 100 drams = $\frac{100 \times 10}{246}$ drams

= 4·065... drams.

2. Since $\frac{95}{100}$ of debt = \$228;

$\therefore \frac{93\frac{1}{2}}{100}$ " $= \$\frac{93\frac{1}{2} \times 228}{\frac{95}{100}}$

$= \$225.$

3. Value of goods on which \$17.50 is duty = \$100 ;

$\therefore$ " " " \$637 "

$= \$\frac{637 \times 100}{17.50}$

$= \$3640.$

4. Since $\frac{12\frac{1}{2}}{100}$ of population of 1870 = 5975 ;

$\therefore$ the " " $= \frac{100 \times 5975}{12\frac{1}{2}}$

$= 47800.$

The population in 1860 is $47800 - 5975 = 41825.$

5. If r represent the rate per cent.,

then $7600 \times (1 + \frac{r}{100})^2 = 9196$

and $(1 + \frac{r}{100})^2 = \frac{9196}{7600} = 1.21 ;$

$\therefore \quad 1 + \frac{r}{100} = \sqrt{1.21} = 1.1 ;$

$\therefore \quad \frac{r}{100} = .1$

and $r = 100 \times .1 = 10.$

II.—Page 198.

1. He had \$1339.60 left out of the part on which he had to pay tax.

Sum from which \$98.50 is left = \$100 ;

$\therefore$ " " \$1339.60 " $= \$\frac{1339.60 \times 100}{98.50}$

$= \$1360 ;$

$\therefore$ his entire salary was $\$(1360 + 400) = \$1760.$

2. Sum expended on bridge = $\frac{97}{100}$ of \$7340

= \$7119.80.

3. Sum of 10 results $= 10 \times 17.5 = 175.$

Sum of first 3 $= 3 \times 16.25 = 48.75.$

Sum of next 4 $= 4 \times 16.5 = 66 ;$

$\therefore$ sum of last 3 $= 175 - 114.75.$
$= 60.25.$

Ninth $=$ tenth $- 1.$
Eighth $=$ tenth $- 4$;
$\therefore$ sum of last 3 $= 3 \times \text{tenth} - 5 = 60.25$;
$\therefore\ 3 \times \text{tenth} = 65.25$;
$\therefore$ tenth $= 21.75.$

4. Since 6 % of gross receipts $= \$42525$;
$\therefore$ the " $= \$\frac{100 \times 42525}{6}$
$= \$708750.$

Now, $3\frac{1}{2}$ % of the capital $= \frac{54}{1000}$ of $\$708750$;
$\therefore$ the paid up capital $= \$\frac{100 \times 54 \times 708750}{3\frac{1}{2} \times 100}$
$= \$10935000.$

5. Part A does in 1 hr. $= \frac{1}{600}$.
" B " $= \frac{3}{1600}$.

Time A and B take to do all $= \frac{1}{\frac{1}{600} + \frac{3}{1600}}$ hr.
$= \frac{4800}{17}$ hr.

Hence A does $\frac{4800}{17} \times \frac{1}{600}$, or $\frac{8}{17}$ of the work,
and B does $\frac{4800}{17} \times \frac{3}{1600}$, or $\frac{9}{17}$ of it.
Cost of $\frac{17}{17}$ of work $= \$85$;
$\therefore$ " $\frac{8}{17}$ " $= \$(8 \times 5) = \$40,$
and " $\frac{9}{17}$ " $= \$(9 \times 5) = \$45.$

III.—Page 198.

1. The cost of a policy of $\$100 = \$(5\frac{1}{4} + \frac{1}{5} + \frac{7}{16})$
$= \$5.8875$;
$\therefore$ policy which covers goods worth $= \$94.1125 = \$100$;
$\therefore$ " " " $\$7905.45$
$= \$\frac{7905.45 \times 100}{94.1125}$
$= \$8400.$

2.

DATE.	Received.	Delivered.	Balance.	Days.	PRODUCTS.
1877.					
January 1.....	2310		2310	15	34650
" 16.....	120		2430	16	38880
February 1.....	300		2730	21	57330
" 22.....		1000	1730	7	12110
March 1.....		600	1130	33	37290
April 3.....		400	730	7	5110
April 10.....		312	418	21	8778
	2730	2312			30)194148
					6471·6

Balance May 1, 418.

6471.6×5 cents $= \$323.58$.

3. $2\frac{1}{2}\% = \frac{10}{400}$ per unit.

$7\frac{1}{2}\% = \frac{30}{400}$ "

$10\frac{1}{4}\% = \frac{41}{400}$ "

The net increase $= \frac{10}{400}$ of males $+ \frac{10}{400}$ of females.

The decrease of males $= \frac{30}{400}$ of males

The increase of females $= \frac{41}{400}$ which must

$\therefore$ = to decrease of males and total net increase;

$\therefore \frac{41}{400}$ of females $= \frac{10}{400}$ of males $+ \frac{10}{400}$ of females $+ \frac{30}{400}$ of males;

or $\frac{31}{400}$ of females $= \frac{40}{400}$ of males;

$\therefore$ the numbers are as 31 to 40.

4. $\frac{105}{100}$ of $\frac{125}{100}$ of single ticket $= \$10.50$;

$\therefore$ cost of single ticket $= \$\frac{100 \times 100 \times 10\,50}{105 \times 125}$

$= \$8$.

5. Paper duty $= 1\frac{1}{2} \times 1\frac{1}{2}d. = 2\frac{1}{4}d.$

Cost of duty to retail dealer $= \frac{120}{100}$ of $\frac{110}{100}$ of $2\frac{1}{4}d.$

$= 2{\cdot}97d.$

IV.—Page 199.

1. Cost of $37\frac{1}{2}$ yd. $= 37\frac{1}{2} \times \$4.87\frac{1}{2} = \$182{\cdot}8125.$
" $49\frac{3}{8}$ yd. $= 49\frac{3}{8} \times \${\cdot}93\frac{3}{4} = \$46{\cdot}5234375.$
Total cost $= \$229{\cdot}3359375.$
Selling price $= \frac{4}{3}$ of $\$229{\cdot}3359375 = \$305.78\frac{1}{8}.$

2. Selling price of cotton less com. $= \frac{98\frac{1}{2}}{100}$ of 12000×7 cents $= \$825.30.$
Sum to be invested in sugar $= \$\frac{825.30 \times 100}{101\frac{1}{2}}.$
Number of pounds bought $= \frac{825.30 \times 100 \times 100}{5 \times 101\frac{1}{2}}$ lb.
$= 16222.11\ldots$ lb.

3. Number that do well $= \frac{22}{100}$ of $750 = 165.$
" barely pass $= \frac{34}{100}$ of $750 = 255.$
" fail $= \frac{44}{100}$ of $750 = 330.$

4. First commission $= \frac{1}{20}$ of sum realized.
Sum to be invested $= \frac{19}{20}$ " "
Second commission $= \frac{1}{51}$ of $\frac{19}{20}$ " "
$= \frac{19}{1020}$ " "
$\therefore$ total commission $= (\frac{1}{20} + \frac{19}{1020})$ " "
$= \frac{70}{1020}$ " "
Hence $\frac{70}{1020}$ of sum realized $= \$70;$
$\therefore$ " " $= \$\frac{1020 \times 70}{70}$
$= \$1020.$
Sum invested in groceries $= \$(1020 - 70) = \$950.$
NOTE.—See also solution 8, Ex. cv., page 59.

5. Taking B's flour as the standard and reducing A's and C's to this standard,
Amount of flour A has of B's standard $= \frac{110}{100}$ of 125 bbl.
$= 137.5$ bbl.
Amount of flour C has of B's " $= \frac{105\frac{5}{11}}{100}$ of $\frac{110}{100}$ of 225 bbl.
$= 261$ bbl.

Selling price of flour $= (125 + 150 + 225) \times \$7$
$= \$3500$
Sum to be remitted $= \frac{96}{100}$ of $\$3500$
$= \$3360.$

He must pay \$3360 to A, B, and C in the proportion of 137.5, 150, and 261.

Hence A receives $\frac{137.5}{548.5}$ of $\$3360 = \$842.30$ (nearly).
B receives $\frac{150}{548.5}$ of $\$3360 = \$918.87$ " .
C receives $\frac{261}{548.5}$ of $\$3360 = \$1598.83$ " .

V.—Page 199.

1. Sum gained, had none proved worthless
$= \$600.$
Cost of \$1 bill $= \$(\cdot 75 + \cdot 01\frac{1}{3}) = 76\frac{1}{3}$ cents.
Sum on which \$.23⅓ is gained $= \$1$;
$\therefore$ " \$600 " " $= \$\frac{600 \times 1}{.23\frac{1}{3}}$.
$= \$2535\frac{5}{7}$

2. Net sum resulting from sale of goods $= \frac{103}{100}$ of \$1910;
$\therefore$ value of goods sold $= \frac{100}{95\frac{1}{4}}$ of $\frac{103}{100}$ of \$1910
$= \$2040.$

3. Sum invested out of \$104 received $= \$100$;
$\therefore$ " " \$30056 "
$= \$\frac{30056 \times 100}{104}$;
$\therefore$ No. of bales bought $= \frac{30056 \times 100}{289 \times 104}$
$= 100$ bales.

4. Sum remitted $= 300 \times \$16.15$
$= \$4845$;
$\therefore$ value of goods sold $= \$\frac{4845 \times 100}{95}$;
and commission $= \frac{5}{100}$ of $\$\frac{4845 \times 100}{95}$
$= \$255.$

5. Cost of \$100 insurance $= 15 \times \$2{\cdot}8674$
$= \$43{\cdot}011.$
Gain on \$100 insurance $= \$56{\cdot}989$;
$\therefore$ insurance on which \$1709.69 is gain
$= \$\frac{1709{\cdot}69 \times 100}{56{\cdot}989}$
$= \$3000$ (nearly).

Examples (cix). Page 203.

1. Gain on \$3.20 $= \${\cdot}80$;
$\therefore$ " \$100 $= \$\frac{100 \times {\cdot}80}{3{\cdot}20}$
$= \$25.$

2. Cost of goods sold for \$112 $= \$100$;
$\therefore$ " " \$2240 $= \$\frac{2240 \times 100}{112}$
$= \$2000.$

3. Cost of 375 yd. $= \$\frac{1960 \times 100}{120}$;
$\therefore$ " 1 yd. $= \$\frac{1960 \times 100}{375 \times 120}$
$= \$4.35.$

4. Desired selling-price of what is sold for $95d. = 115d$
$\therefore$ " " " " $209d.$
$= \frac{209 \times 115}{95}d.$
$= £1\ 1s.\ 1d.$

5. Cost price $= \$\frac{544 \times 100}{84} = \$647\frac{13}{21}$.
Hence gain on $\$647\frac{13}{21} = \$24\frac{8}{21}$;
$\therefore$ " " \$100 $= \$\frac{100 \times 24\frac{8}{21}}{647\frac{13}{21}}$
$= \$3\frac{13}{17}$

6. Gain on $1\frac{1}{2}d. = \frac{1}{2}d.$
$\therefore$ " $100d. = \frac{100 \times \frac{1}{2}}{1\frac{1}{2}}d.$
$= 33\frac{1}{3}d.$

7. Sum charged $= \frac{90}{100}$ of $\frac{120}{100}$ of cost
$= \frac{108}{100}$ of cost;
$\therefore$ he gains $\frac{8}{100}$ of cost, and hence his gain per cent.
$= 8$.

8. Sum charged $= \frac{88}{100}$ of $\frac{125}{100}$ of cost
$= \frac{110}{100}$ of cost;
$\therefore$ he gains 10 %.

9. Loss on an outlay of £23 5*s*. 5*d*. = £1 3*s*. $3\frac{1}{4}d$.;
$\therefore$ " " £100 $= \frac{100 \times (£1\ 3s.\ 3\frac{1}{4}d.)}{23\frac{13}{48}}$
$= £5$.

10. Loss on an outlay of £15 6*s*. 3*d*. = £3 10*s*. $5\frac{1}{4}d$.
$\therefore$ " " £100 $= \frac{100 \times (£3\ 10s.\ 5\frac{1}{4}d.)}{15\frac{5}{16}}$
$= £23$.

11. Cost of 54.87 cwt. = 54.87 × \$96 = \$5267.52
$\therefore$ Gain on \$5267.52 = \$732.48
$\therefore$ " \$100 $= \$\frac{100 \times 732.48}{5267.52}$
$= \$13.9....$

12. Cost price $= \$\frac{4125.60 \times 100}{108}$
$= \$3820$.
Gain on \$3820 = \$(4202 − 3820) = \$382;
$\therefore$ " \$100 $= \$\frac{100 \times 382}{3820}$
$= \$10$.

13. 3 % of original price = \$9;
$\therefore$ the original price $= \$\frac{100 \times 9}{3}$
$= \$300$.

14. Cost of 12 lb. = 12 × 2*s*. $6\frac{1}{2}d$. = 30*s*. 6*d*.
" 4 lb. = 4 × 3*s*. $2\frac{1}{4}d$. = 12*s*. 9*d*.
Total cost 43*s*. 3*d*.
Selling price of 1 lb. $= \frac{\frac{4}{3} \times 43s.\ 3d.}{16}$
$= 3s.\ 7\frac{1}{4}d.$

15. Cost of 1 lb. of mixture $= \$\frac{1.56\frac{1}{4}\times 100}{133\frac{1}{3}}$
$= \$1.17\frac{1}{2}$.

Now $\$\frac{(1.05+1.30)}{2} = \$1.17\frac{1}{2}$;

$\therefore$ he must have the same quantity of each kind.

16. Cost of 80 gal. $= 80\times\$3.60 = \$288$.
" 180 gal. $= 180\times\$3.00 = \$540$.
Selling price of 1 gal. $= \$\frac{828\times 108\frac{1}{3}}{240\times 100} = \$3.45$.

17. Cost of 80 gal. $= 80\times\$3.10 = \$248$.
" 96 gal. $= 96\times\$3.41\frac{2}{3} = \$328$.
Selling price of 1 gal. $= \$\frac{576\times 110}{176\times 100} = \$3.60$.

18. Cost of 3 lb. at $61\frac{2}{3}$ ct. $= \$1.85$.
" 1 lb. at 55 ct. $= \$.55$;
$\therefore$ cost of 1 lb. of mixture $= \$\frac{2.40}{4}$
$= 60$ ct.
Gain on an outlay of 60 ct. $= 20$ ct.;
$\therefore$ " " 100 ct. $= \frac{100\times 20}{60}$ ct.
$= 33\frac{1}{3}$ ct.

Examples (cx.) Page 212.

36. Sum paid for an income of $\$6 = \$100$;
$\therefore$ " " " " $\$5 = \$\frac{5\times 100}{6}$
$= \$83\frac{1}{3}$.

37. An investment of \$125 yields \$9 income;
$\therefore$ " " \$100 " $\$\frac{100\times 9}{125}$ income
$= \$7\frac{1}{2}$.
Again an investment of \$75 yields \$6 income;
$\therefore$ " " " \$100 " $\$\frac{100\times 6}{75}$ income
$= \$8$.
$\therefore$ the second is more advantageous by $\frac{4}{5}$ %.

38. Income from £1 in the 1st stock $= £1\frac{6}{126} = £\frac{1}{21}$.
Income from £1 " 2nd " $= £2\frac{9}{10} = £\frac{3}{70}$.
Sum invested for difference of income of $£(\frac{1}{21} - \frac{3}{70})$
$= £1$;
∴ " " " " " £22½
$= £\frac{22\frac{1}{2} \times 1}{\frac{1}{21} - \frac{3}{70}}$
$= £4725.$

39. Income from £96 invested in 3 per cents.
$= £3.$
" " £96 " in R.R. stocks $= £\frac{96 \times 5}{100}$
$= £4.8$;
∴ the income is increased £1.8.

40. Net income on £91 invested $= £(3\frac{1}{2} - 2\frac{7}{240})$
$= £\frac{833}{240}.$
Sum invested for an income of $£\frac{833}{240} = £91$;
∴ " " " " " " £952 $= £\frac{952 \times 91}{\frac{833}{240}}$
$= £24960.$

41. Money from sale of £4500 stock $= £\frac{4500 \times 112.5}{100}$.
First income $= £\frac{4500 \times 5}{100} = £225.$
Second " $= £(225 + 168\frac{3}{4}) = £393\frac{3}{4}.$
Amount of Egyptian stock $= \frac{393\frac{3}{4} \times 100}{7}$
$= £5625.$
Sum paid for £5625 stock $= £(45 \times 112.5)$;
∴ " " £100 stock $= £\frac{100 \times 45 \times 112.5}{5625}$
$= £90.$

42. Money from sale of £3200 stock $= £\frac{3200 \times 86\frac{1}{4}}{100}$
$= £2760.$
First income $= £\frac{3200 \times 3}{100} = £96.$

Income from investing £115 $= £4$;

$\therefore$ " " £2760 $= £\frac{2760 \times 4}{115}$

$= £96$;

$\therefore$ no alteration is made.

43. Income from investing $\$105\frac{1}{2} = \$6$;

$\therefore$ $\therefore$ " $\$8229 = \$\frac{8229 \times 6}{105\frac{1}{2}}$

$= \$468$;

$\therefore$ it is increased by $\$(468 - 411.45)$

$= \$56.55.$

44. Income from investing \$108 in 1st stock

$= \$6$

" " \$216 in 2nd "

$= \$\frac{216 \times 5}{99}$

$= \$10\frac{10}{11}$

Sum invested for income of $\$16\frac{10}{11} = \$324$;

$\therefore$ " " $\$1674 = \$\frac{1674 \times 324}{16\frac{10}{11}}$

$= \$32076.$

Sum invested in 1st stock $= \$\frac{32076}{3}$

$= \$10692.$

" " " 2nd " $= 2 \times \$10692$

$= \$21384.$

45. Value of \$5500 in currency $= \$\frac{5500 \times 125}{100}$

$= \$6875$;

$\therefore$ he will gain $\$(7000 - 6875)$, or \$125 in currency.

46. On an investment of \$92 I lose 7.

Income from investing $\$85 = \$\frac{85 \times 5}{100}$

$= \$4\frac{1}{4}$;

$\therefore$ sum gained each year $= \$(4\frac{1}{4} - 3)$

$= \$1\frac{1}{4}$;

hence number of years $= \frac{7}{1\frac{1}{4}}$ yr. $= 5\frac{3}{5}$ yr.

47. Money received from sale $= \$\frac{4000 \times 179\frac{3}{5}}{100}$.

Original income $= \$\frac{4000 \times 14}{100}$

$= \$560.$

New income $= \$\frac{4000 \times 179\frac{3}{5} \times 9}{100 \times 125\frac{3}{5}}$

$= \$514.77\ldots;$

$\therefore$ my loss $= \$45.22\ldots.$

48. Loss on each \$100 of stock bought $= \$2.875.$

He gained the dividend at $3\frac{1}{2}\ \% = \$3.50;$

$\therefore$ amount of stock from which the net profit is

$\$.625 = \$100;$

$\therefore$ " " " $\$37.50$

$= \$\frac{37.50 \times 100}{.625}$

$= \$6000.$

49. Profit from investing \$1 in 6 per cents

$= \$\frac{6}{99} = \$\frac{1}{16\frac{1}{2}}.$

" " " 5 per cents

$= \$\frac{5}{85\frac{1}{2}} = \$\frac{1}{17\frac{1}{10}};$

hence the former is preferable.

50. Gross income $= \$\left(\frac{1375.50 \times 100}{98\frac{1}{4}} + 400\right)$

$= \$1800.$

(See Ex. paper II. ex. 1, page 62.)

Investment to yield an income of \$6 $= \$101;$

$\therefore$ " " " " \$1800 $= \$\frac{1800 \times 101}{6}$

$= \$30300.$

51. Since 3% of his stock $= £2400$;

$$\therefore \text{his stock} = £\frac{100 \times 2400}{3}$$
$$= £80000.$$

$$\text{Cost of £8000 stock} = £\frac{80000 \times 94\frac{1}{8}}{100}$$
$$= \$(800 \times 94\tfrac{3}{8} \times 4.86\tfrac{2}{3});$$

$$\therefore \text{income in Canada} = \$\frac{800 \times 94\frac{3}{8} \times 4.86\frac{2}{3} \times 12}{100}$$
$$= \$44092.$$

52. Money from sale of £4500 stock $= £\frac{4500 \times 112.5}{100}$
$= £(45 \times 112.5).$

First income $= £\frac{4500 \times 5}{100} = £225.$

Second income $= £(225 + 168\frac{3}{4}) = £393.75$

Amount of Egyptian stock $= £\frac{393.75 \times 100}{7}$
$= £5625.$

Sum paid for £5625 stock $= £(45 \times 112.5)$

$\therefore$ " " £100 stock $= £\frac{100 \times 45 \times 112.5}{5625}$
$= £90.$

Hence the market price of stock $= £(90 - \frac{1}{4})$
$= £89\frac{3}{4}.$

53. Amount of stock bought in the 6's

$$= \$\frac{\text{Sum} \times 100}{91\frac{1}{2}}$$

Amount of stock bought in the 7's

$$= \$\frac{\text{Sum} \times 100}{102};$$

$$\therefore \$\frac{\text{sum} \times 100}{91\frac{1}{2}} - \$\frac{\text{sum} \times 100}{102} = \$3500;$$

$$\therefore \text{sum} \times (\tfrac{1}{91\frac{1}{2}} - \tfrac{1}{102}) = \$35$$

$$\text{and sum} = \$\frac{35}{\frac{2}{183} - \frac{1}{102}}$$
$$= \$(5 \times 61 \times 102).$$

Income from investing in the 6's $= \frac{5\times61\times102\times6}{91}$

$= \$2040.$

Income from investing in the 7's $= \$\frac{5\times61\times102\times7}{102}$

$= \$2135;$

$\therefore$ difference of income $= \$95.$

54. Income from $\$124.5 = \$5;$

$\therefore$ " $\$100 = \$\frac{100\times5}{124.5}$

$= \$4\frac{4}{249}.$

Income from $\$(\frac{100}{30}\times34) = \$1\frac{5}{6};$

$\therefore$ " $\$100 = \$\frac{100\times4\frac{5}{6}}{\frac{100\times34}{30}}$

$= \$4\frac{9}{34}.$

55. Since $3\frac{3}{4}$ % of the capital

= 3 % of (capital $-\$1200000$) + 5 % of $\$1200000$;

$\therefore$ $\frac{3}{4}$ % of the capital = 5 % of $\$1200000$ − 3 % of $\$1200000$,

and $\frac{3}{4}$ % of the capital = $\$24000,$

and capital $= \$\frac{400\times24000}{3}$

$= \$3200000.$

56. Sum to be invested $= \$\frac{6150\times100}{102\frac{1}{2}}.$

Value of this sum in gold $= \$\frac{6150\times100\times100}{102\frac{1}{2}\times115}.$

Number of yards bought $= \frac{6150\times100\times100}{1.03\frac{1}{2}\times102\frac{1}{2}\times115}$

$= 5040\frac{1520}{1587}$ yd.

EXAMINATION PAPERS.

I.—Page 215.

1. Desired sale of goods worth $\$91 = \$107;$

$\therefore$ " " " $3\times\$728$

$= \$\frac{3\times728\times107}{91}$

$= \$2568.$

2. Cost price of article $= \$\frac{2.10 \times 100}{120}$
$= \$1.75.$
Loss on \$1.75 $= \$.15$;
$\therefore$ " \$100 $= \$\frac{100 \times .15}{1.75}$
$= \$8\frac{4}{7}.$

3. Cost price per lb. $= \$\frac{1.80 \times 100}{107\frac{1}{2}}.$
Entire cost of tea $= \$\frac{150 \times 1.80 \times 100}{107\frac{1}{2}}.$
Entire selling price $= \$\frac{150 \times 1.80 \times 100 \times 110}{107\frac{1}{2} \times 100}$
$= \$276\frac{12}{43}.$
Selling price of 50 lb. $= \$90$;
$\therefore$ " " 100 lb. $= \$186\frac{12}{43}$;
$\therefore$ " " 1 lb. $= \$1.86\frac{12}{43}.$

4. Marked selling price $= \frac{135}{100}$ of cost price.
Real " $= \frac{90}{100}$ of $\frac{135}{100}$ of cost price
$= \frac{1214}{100}$ of cost price;
$\therefore$ his net gain $= \frac{214}{100}$ " "
$= 21\frac{1}{2}\ \%.$

5. Sum required to take up the bill $= \$\frac{2520 \times 100}{105}$
$= \$2400.$
Interest on \$2400 each quarter $= \$(2400 \times \frac{1}{4} \times \frac{9}{200})$
$= \$27.$
Amount of \$27 for 3 payments $= 27 \times \$(1\frac{9}{800})^3$
" " 2 " $= 27 \times \$(1\frac{9}{800})^2$
" " 1 payment $= 27 \times \$(1\frac{9}{800}).$
Total interest received
$= \$\{27 + 27 \times (1\frac{9}{800}) + 27 \times (1\frac{9}{800})^2 + 27 \times (1\frac{9}{800})^3\}$
$= 27 \times \$\{(1 + 1\frac{9}{800}) + (1\frac{9}{800})^2 + (1\frac{9}{800})^3\}$
$= 27 \times \$\{(1 + 1{\cdot}01125 + 1{\cdot}02262\ldots + 1{\cdot}03413\ldots)\}$
$= 27 \times \$4{\cdot}068\ldots$
$= \$109{\cdot}836\ldots$

Sum realized from \$2400 = \$2509·836... ;
∴ his loss would = \$(2520 − 2509·836...)
= \$10·163...

II.—Page 216.

1. Present worth of \$2.45 = $\$\frac{2.45 \times 100}{105}$
= \$2.33⅓.

2 Conditional price = $\frac{95}{100}$ of selling price.
Amount of \$100 for 3 mo. = \$101¼ ;
∴ actual selling price 3 mo. before
= $\frac{100}{101\frac{1}{4}}$ of $\frac{90}{100}$ of selling price.
= $\frac{88\frac{8}{9}}{100}$ of selling price ;
∴ discount allowed = $\frac{11\frac{1}{9}}{100}$ " "
= 11⅑ %.

Again, actual selling price 3 mo. after
= $\frac{101\frac{1}{4}}{100}$ of $\frac{90}{100}$ of selling price
= $\frac{91\frac{1}{8}}{100}$ of selling price ;
∴ discount allowed = $\frac{8\frac{7}{8}}{100}$ " "
= 8⅞ %.

3. In 1 oz. avoir. weight there are $\frac{7000}{16}$ gr.
Cost of 5760 gr. = \$1.20 ;
∴ " $\frac{7000}{16}$ gr. = $\$\frac{\frac{7000}{16} \times 1.20}{5760}$
= 9$\frac{11}{96}$ cents.

4. See Art. 198.

Price of £10000 stock = £9000;

∴ " £100 stock = £90.

5. Money got from sale = $\$\frac{1200\times 86}{100}$

= \$1032.

Income from 3 per cents = $\$\frac{1200\times 3}{100}$

= \$36;

∴ price of 8 % stock = $\$\frac{1032\times 8}{36}$

= $\$229\frac{1}{3}$.

III.—Page 216.

1. Sum invested = $\$\frac{3060\times 100}{102}$

= \$3000.

Number of pounds bought = $\frac{3000}{.75}$ = 4000 lb.

Total cost of 4000 lb. = \$(3060 + 30 + $\frac{1}{300}$ of 3000)

= \$3100;

∴ selling price of 4000 lb. = $\$\frac{3100\times 140}{100}$;

∴ " " 1 lb. = $\$\frac{3100\times 140}{4000\times 100}$

= $\$1.08\frac{1}{2}$.

2. Selling price = $\$\frac{50\times 120}{100}$

= \$60.

But \$60 is only $\frac{75}{100}$ of asking price;

∴ asking price = $\$\frac{60\times 100}{75}$

= \$80.

3. Present worth of \$2.25 = $\$\frac{2.25\times 100}{105}$

= $\$2.14\frac{2}{7}$.

Hence *A* buys at the lower rate.

Marking price of *A*'s silk = $\$\frac{2.14\frac{2}{7}\times 125}{100}$

= $\$2.67\frac{6}{7}$.

Marking price of B's silk $= \$\frac{2.15 \times 125}{100}$

$= \$2.68\frac{3}{4}$.

Gain on an outlay of $\$2.14\frac{2}{7} = \$.85\frac{5}{7}$;

$\therefore$ " " " $\$100 = \$\frac{85\frac{5}{7}}{2.14\frac{2}{7}}$

$= \$40$.

Gain on an outlay of $\$2.15 = \$.85$

$\therefore$ " " " $\$100 = \$\frac{85}{2.15}$

$= \$39\frac{23}{43}$.

4. Supposed cost price $= \frac{19}{20}$ of cost price.

Supposed selling price $= \frac{21}{20}$ of cost price.

Then $\frac{21}{20}$ of cost price $- \$\frac{1}{20} = \frac{19}{20}$ of $\frac{19}{20}$ of cost price;

and $(\frac{21}{20} - \frac{209}{200})$ of cost price $= \$\frac{1}{20}$;

$\therefore \frac{1}{200}$ of cost price $= \$\frac{1}{20}$,

and cost price $= \$\frac{200}{20}$

$= \$10$.

5. The first payment of interest is \$6, and will be due in 1 yr.; its amount for 2 yrs. will be $\$6(1.05)^2$; similarly, the amount of the second payment will be $\$6(1.05)$; and the amount of the third payment will be \$6. Hence, if P represent the present value of the bonds, we have

$$P(1.05)^3 = 100 + 6(1.05)^2 + 6(1.05) + 6$$

$$= 118.9150;$$

$$\therefore P = \$102.723\ldots.$$

IV.—Page 217.

1. Value of \$4 currency in gold $= \$\frac{1 \times 100}{118}$

$= \$3\frac{23}{59}$.

Gain on \$3 $= \$\frac{23}{59}$;

$\therefore$ " \$100 $= \$\frac{100 \times \frac{23}{59}}{3}$

$= \$12.99\ldots.$

2. Selling price of cheese $= 24 \times \$30 = \$720$.

Cost " 12 cheese $= \$\frac{12\times30\times100}{130}$

$= \$276\frac{12}{13}$

Cost " 12 cheese $= \$\frac{12\times30\times100}{70}$

$= \$514\frac{2}{7}$;

$\therefore$ total cost $= \$791\frac{19}{91}$;

$\therefore$ net loss $= \$71\frac{19}{91}$.

3. Asking price $= \frac{136}{100}$ of cost price.

Selling price $= \frac{80}{100}$ of $\frac{136}{100}$ of cost price

$= \frac{1088}{1000}$ of cost price.

Hence $\frac{1088}{1000}$ of cost price $-$ cost price $= \$528$;

$\therefore$ cost price $= \$\frac{1000\times528}{88}$

$= \$6000$.

Asking price $= \frac{136}{100}$ of $\$6000 = \$8160$.

Selling price $= \frac{80}{100}$ of $\$8160 = \$6528$.

4. If S represent the sum first invested,

every \$73 invested will give \$3 interest;

$\therefore$ " \$1 " " $\$\frac{3}{73}$ "

$\therefore$ " \$S " " $S\times\$\frac{3}{73}$ "

and this interest, $S\times\$\frac{3}{73}$, invested, will give

$S\times\frac{3}{73}\times\$\frac{3}{73}$ interest.

Thus at the end of 2 years there was on hand the first investment and its 2 years' interest, also the interest on the first year's interest, also a second investment of S and one year's interest on it to meet the debt of \$1085.

Hence $(S+2\times S\times\frac{3}{73})+(S\times\frac{3}{73}\times\frac{3}{73})+(S+S\times\frac{3}{73})$

$= \$1085$;

$\therefore S\times\frac{11324}{5329} = \$1085$;

$\therefore S = \$\frac{1085\times5329}{11324}$

$= \$510.59....$

5. Net capital Jan. 1, 1875 $= \$(40000+1750-9350)$
$= \$32400.$
Net capital Jan. 1, 1876 $= \$(39750+2850-7550)$
$= \$35050.$
Amount of \$32400 at 5% $= \$34020.$
Gross profit $= \$(35050+1500-34020)$
$= \$2530.$

V.—Page 218.

1. The dividend $= \frac{8}{100}$ of stock.
Amount of new stock purchased
$= \frac{100}{80}$ of $\frac{8}{100}$ of stock
$= \frac{1}{10}$ of stock.
Hence $\frac{11}{10}$ of stock $= \$13750,$
and stock $= \$12500;$
$\therefore$ the dividend $= \frac{8}{100}$ of \$12500
$= \$1000.$

2. Cost of \$100 of stock $= \$76\frac{2}{3}.$
Selling price " " $= \$82\frac{3}{4};$
$\therefore$ gain on " " $= \$6\frac{1}{12}.$
Amount of stock to gain $\$6\frac{1}{12} = \$100;$
$\therefore$ " " $\$121.66\frac{2}{3} = \$\frac{121.66\frac{2}{3}\times 100}{6\frac{1}{12}}$
$= \$2000;$
$\therefore$ number of shares $= \frac{2000}{50} = 40.$

3. Value of \$400 U. S. currency in gold
$= \$\frac{400\times 100}{175}$
$= \$\frac{1600}{7}$
Sum from which $\$2\frac{1}{2}$ is half yearly dividend $= \$100;$
$\therefore$ " " $\$\frac{1600}{7}$ " "
$= \$\frac{\frac{1600}{7}\times 100}{2\frac{1}{2}}$
$= \$9142\frac{6}{7}.$

4. Whole sum to be collected = \$1700000.
Sum already " = \$1050000.
Sum to be " = \$650000.
Percentage which \$650000 is of \$1500000
$= \frac{100 \times 650000}{1500000}$
$= 43\frac{1}{3}$.

5. Amount of stock bought = $\$\frac{16380 \times 100}{91}$
= \$18000.
Money from \$12000 stock = $\$\frac{12000 \times 93.5}{100}$
= \$11220.
Money from \$6000 stock = $\$\frac{6000 \times 85}{100}$
= \$5100.
Money from both sales = \$16320;
$\therefore$ loss = \$60.
Orignal income = $\$\frac{18000 \times 3}{100}$
= \$540.
New Income = $\$\frac{16320 \times 4.5}{102}$
= \$720.
Hence increase = \$180.

Examples (cxi). Page 220.

3 $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{77}{60}$.

1st share $= \frac{\frac{1}{2}}{\frac{77}{60}}$ of \$8470 = \$3300.

2nd share $= \frac{\frac{1}{3}}{\frac{77}{60}}$ of \$8470 = \$2200.

3rd share $= \frac{\frac{1}{4}}{\frac{77}{60}}$ of \$8470 = \$1650.

4th share $= \frac{\frac{1}{5}}{\frac{77}{60}}$ of \$8470 = \$1320

4. $75 + 10 + 15 = 100$.

Amount of Saltpetre $= \frac{75}{100}$ of 1200 lb. $= 900$ lb.

Amount of Sulphur $= \frac{10}{100}$ of 1200 lb. $= 120$ lb.

Amount of Charcoal $= \frac{15}{100}$ of 1200 lb. $= 180$ lb.

5. $3 + 4 + 5 = 12$.

Length of 1st side $= \frac{3}{12}$ of 480 yd. $= 120$ yd.

" 2nd " $= \frac{4}{12}$ of 480 yd. $= 160$ yd.

" 3rd " $= \frac{5}{12}$ of 480 yd. $= 200$ yd.

6. Representing B's share by 1,

A's share will be 3,

C's share will be 4 ;

$\therefore$ all the shares $= 8$ times B's share.

8 times B's share $= \$640$.

B's " $= \$\frac{640}{8} = \$80$.

A's " $= 3 \times \$80 = \$240$.

C's " $= 4 \times \$80 = \$320$.

7. When the second receives 8 apples, the first receives 7 and the third 10.

$7+8+10 = 25$.

Share of 1st $= \frac{7}{25}$ of $100 = 28$.

" 2nd $= \frac{8}{25}$ of $100 = 32$.

" 3rd $= \frac{10}{25}$ of $100 = 40$.

8. $5450s. + 7085s. + 9810s. = 22345s.$

A gets $\frac{5450}{22345}$ of £418 19$s.$ 4½$d.$ $=$ £102 3$s.$ 9$d.$

B " $\frac{7085}{22345}$ of £418 19$s.$ 4½$d.$ $=$ £132 16$s.$ 10½$d.$

C " $\frac{9810}{22345}$ of £418 19$s.$ 4½$d.$ $=$ £183 18$s.$ 9$d.$

9. $4150 + 12450 + 24900 + 29050 = 70550$.

Share of 1st town $= \frac{4150}{70550}$ of $1921 = 113$.

" 2nd " $= \frac{12450}{70550}$ of $1921 = 339$.

" 3rd " $= \frac{24900}{70550}$ of $1921 = 678$.

" 4th " $= \frac{29050}{70550}$ of $1921 = 791$

10. $10s. + 5s. + 2\frac{1}{2}s. + 1s. + 6d. + 4d. = 232d.$;

$\therefore$ Number of each $= \frac{29 \times 20 \times 12}{232} = 30.$

11. £(500+350+800+90) = £1740.

Share of 1st $= \frac{500}{1740}$ of 200 a. $= 57\frac{41}{87}$ a.

" 2nd $= \frac{350}{1740}$ of 200 a. $= 40\frac{20}{87}$ a.

" 3rd $= \frac{800}{1740}$ of 200 a. $= 91\frac{83}{87}$ a.

" 4th $= \frac{90}{1740}$ of 200 a. $= 10\frac{30}{87}$ a.

12. If B gets $1s.$, A gets $9d.$, and C $2s.$

$1s. + 9d. + 2s. = 45d.$

A's share $= \frac{9}{45}$ of $45s. = 9s.$

B's " $= \frac{12}{45}$ of $45s. = 12s.$

C's " $= \frac{24}{45}$ of $45s. = 24s.$

13. The pay of 7 women = the pay of 3 men.

" 14 boys = " of $\frac{28}{5}$ women.

= " $\frac{12}{5}$ men.

5 men + 3 men + $\frac{12}{5}$ men = $\frac{52}{5}$ men.

Share of the men $= \frac{5}{\frac{52}{5}}$ of \$10.40 = \$5.

" women $= \frac{3}{\frac{52}{5}}$ of \$10.40 = \$3.

" boys $= \frac{\frac{12}{5}}{\frac{52}{5}}$ of \$10.40 = \$2.40.

14. Since there are 9 women, there must be 6 men and 15 children.

But the share of 9 women = share of 6 men

and " 15 children = " 5 men

6 + 6 + 5 = 17.

Share of men $= \frac{6}{17}$ of \$517.65 = \$182.70.

" women $= \frac{6}{17}$ of \$517.65 = \$182.70.

" children $= \frac{5}{17}$ of \$517.65 = \$152.25.

15. $20 + 18 + 12 = 50$

Share of youngest $= \frac{12}{50}$ of property $= \$1440$;

$\therefore$ the value of the property $= \frac{50 \times 1440}{12}$

$= \$6000.$

16. Take B's share as the unit,

then C's " $= \frac{2}{5}$ of B's $+ \$800$,

and A's " $= \frac{5}{9}$ of $(\frac{2}{5}$ of B's $+ \$800) - \$300$;

Sum of all the shares $= \frac{73}{45}$ of B's $+ \$944\frac{4}{9}$;

$\therefore \frac{73}{45}$ of B's $+ \$944\frac{4}{9} = \$5000$;

$\frac{73}{45}$ of B's $= \$4055\frac{5}{9}$;

B's $= \$2500.$

C's $= \frac{2}{5}$ of $\$2500 + \$800$

$= \$1800.$

A's $= \frac{5}{9}$ of $\$1800 - \$300$

$= \$700.$

17. Take D's share as the unit.

C's " $= \frac{9}{10}$ of D's $- \$100.$

B's " $= \frac{4}{5}$ of $(\frac{9}{10}$ of D's $- \$100) + \$200$

$= \frac{36}{50}$ of D's $+ \$120.$

A's " $= \frac{3}{4}$ of $(\frac{36}{50}$ of D's $+ \$120) + \$250.$

Sum of all the shares $= \frac{155}{50}$ of D's $+ \$350.$

$\therefore \frac{155}{50}$ of D's $+ \$350 = \$5000$;

$\frac{31}{10}$ of D's $= \$4650$;

D's $= \$1500.$

C's $= \frac{9}{10}$ of $\$1500 - \$100$

$= \$1250.$

B's $= \frac{4}{5}$ of $\$1250 + \$200$

$= \$1200.$

A's $= \frac{3}{4}$ of $\$1200 + \$250$

$= \$1050.$

18. Take the first fraction as the unit,
then the second $= \frac{22}{23}$ of the first,
and the third $= \frac{22}{24}$ " "
Sum of the 3 fractions $= \frac{1586}{552}$ " "
$\therefore \frac{1586}{552}$ of the first $= \frac{183}{242}$,

$$\text{and first} = \frac{552 \times 183}{1586 \times 242} = \frac{414}{1573};$$
$$\text{second} = \frac{22}{23} \text{ of } \frac{414}{1573} = \frac{36}{143};$$
$$\text{third} = \frac{22}{24} \text{ of } \frac{414}{1573} = \frac{69}{286}.$$

19. Simple interest $= \$\frac{1171 \times 13 \times 6}{100}$
$= \$913.38.$
$\frac{5}{8} + \frac{7}{9} + \frac{9}{10} + \frac{5}{12} + \frac{8}{15} = \frac{1171}{360}.$

Share of 1st $= \frac{\frac{5}{8}}{\frac{1171}{360}}$ of \$913.38 $=$ \$175.50.

" 2nd $= \frac{\frac{7}{9}}{\frac{1171}{360}}$ of \$913.38 $=$ \$218.40.

" 3rd $= \frac{\frac{9}{10}}{\frac{1171}{360}}$ of \$913.38 $=$ \$252.72.

" 4th $= \frac{\frac{5}{12}}{\frac{1171}{360}}$ of \$913.38 $=$ \$117.00.

" 5th $= \frac{\frac{8}{15}}{\frac{1171}{360}}$ of \$913.38 $=$ \$149.76.

20. $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$;
$\therefore$ the boys over 15 get $\frac{1}{2}$ of \$400
$=$ \$200;
the boys between 10 and 15 get $\frac{1}{3}$ of \$400
$=$ \$133$\frac{1}{3}$;
and the rest get $\frac{1}{6}$ of \$400
$=$ \$66$\frac{2}{3}$.

Number of boys over $15 = 200 \times 2 = 400$;
$\therefore$ " " in school $= 3 \times 400 = 1200.$

Examples (cxii). Page 222.

1. Rent of a house worth \$2592 = \$132.75;

$\therefore$ " \$864 = $\$\frac{864 \times 132.75}{2592}$

= \$44.25;

$\therefore$ " \$1728 = $\$\frac{1728 \times 132.75}{2592}$

= \$88.50.

2. 18 cows eat as much as 9 horses;
and 90 sheep " " 15 horses.

$6 + 9 + 15 = 30.$

A's share $= \frac{6}{30}$ of \$22.50 = \$4.50.

B's " $= \frac{9}{30}$ of \$22.50 = \$6.75.

C's " $= \frac{15}{30}$ of \$22.50 = \$11.25.

3. Profit to be divided $= \frac{1}{5}$ of \$25780 = \$5156.

A contributed $\frac{2}{5}$ of capital; B, $\frac{9}{20}$ of capital and C, $\frac{3}{20}$ of it.

A's profit $= \frac{2}{5}$ of \$5156 = \$2062.40.

B's " $= \frac{9}{20}$ of \$5156 = \$2320.20.

C's " $= \frac{3}{20}$ of \$5156 = \$773.40.

4. A's money was in the business 287 days and B's 167 days.

$287 \times 2400 = 688800$

$167 \times 1800 = 300600$

989400

$\therefore$ A's share $= \frac{688800}{989400}$ of \$943

$= \$656\frac{820}{1649}$;

and B's share $= \frac{300600}{989400}$ of \$943

$\$286\frac{820}{1649}$.

5. $3 \times 40 = 120$

$4 \times 75 = 300$

$420.$

D's share $= \frac{120}{420}$ of \$70 = \$20.
E's " $= \frac{300}{420}$ of \$70 = \$50.

6. $7 \times 500 = 3500$
$8 \times 600 = 4800$
$9 \times 900 = 8100.$
$16400.$

A's share $= \frac{3500}{16400}$ of \$410 = \$87.50.
B's " $= \frac{4800}{16400}$ of \$410 = \$120.
C's " $= \frac{8100}{16400}$ of \$410 = \$202.50.

7. $3 \times 10 = 30$
$4 \times 12 = 48$
$2 \times 14 = 28$
$106.$

Share of 1st $= \frac{30}{106}$ of \$106 = \$30.
" 2nd $= \frac{48}{106}$ of \$106 = \$48.
" 3rd $= \frac{28}{106}$ of \$106 = \$28.

8. First works $(6 \times 9 + 6 \times 8)$ hrs. = 102 hrs.
Second " $10 \times 9\frac{1}{2}$ hrs. = 95 "
197

Share of 1st $= \frac{102}{197}$ of \$29.55 = \$15.30.
Share of 2nd $\frac{95}{197}$ of \$29.55 = \$14.25.

9. $12 \times 400 = 4800$
$10 \times 500 = 5000$ } 9800 = *A's* capital for 1 mo.

$12 \times 300 = 3600$
$9 \times 600 = 5400$ } 9000 = *B's* " " "
18800.

A's share $= \frac{9800}{18800}$ of \$470 = \$245.
B's " $= \frac{9000}{18800}$ of \$470 = \$225

10. $12\times3000 = 36000$
$12\times4500 = 54000$ $\Big\}$ $= 90000.$
$15\times3500 = 52500$
$9\times2500 = 22500$ $\Big\}$ $= 75000.$
$12\times2250 = 27000$ $= 27000.$
$192000.$

Johnston's share $= \frac{90000}{192000}$ of \$1248
$= \$585.$
Wilson's share $= \frac{75000}{192000}$ of \$1248
$= \$487.50.$
Miller's share $= \frac{27000}{192000}$ of \$1248
$= \$175.50.$

11. $1\frac{1}{2}\times10 = 15$ horses $= 45$ sheep
$2\times30 = 60$ oxen $= 120$ "
$3\frac{1}{4}\times100 = 325$ sheep $= 325$ " $\Big\}$ $= 490.$
$2\frac{1}{2}\times40 = 100$ horses $= 300$ sheep
$1\frac{1}{4}\times50 = 62\frac{1}{2}$ oxen $= 125$ "
$3\times115 = 345$ sheep $= 345$ " $\Big\}$ $= 770.$
$1260.$

A's share $= \frac{490}{1260}$ of \$88.20 $= \$34.30.$
B's " $= \frac{770}{1260}$ of \$88.20 $= \$53.90.$

12. $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5} = \frac{77}{60}.$

As there are only $\frac{60}{60}$ in his property he could not possibly leave $\frac{77}{60}$.

A's share $= \frac{\frac{1}{2}}{\frac{77}{60}}$ of \$1886.50 $= \$735.00.$

B's " $= \frac{\frac{1}{3}}{\frac{77}{60}}$ of \$1886.50 $= \$490.00.$

C's " $= \frac{\frac{1}{4}}{\frac{77}{60}}$ of \$1886.50 $= \$367.50.$

D's " $= \frac{\frac{1}{5}}{\frac{77}{60}}$ of \$1886.50 $= \$294.00.$

13. $36 + 54 + 78 = 168$.

A's $= \frac{36}{168}$ of 78 gal. $= 16\frac{5}{7}$ gal.

B's $= \frac{54}{168}$ of 78 gal. $= 25\frac{1}{14}$ gal.

14. A uses the *whole* house for 4 mo.; half of it for $5\frac{1}{2}$ mo., and $\frac{1}{3}$ of it for $2\frac{1}{2}$ mo.

B uses $\frac{1}{2}$ the house for $5\frac{1}{2}$ mo., and $\frac{1}{3}$ of it for $2\frac{1}{2}$ mo.

C uses $\frac{1}{3}$ the house for $2\frac{1}{2}$ mo.

$$\left.\begin{array}{l} 4 \times 1 = 4 \\ 5\frac{1}{2} \times \frac{1}{2} = 2\frac{3}{4} \\ 2\frac{1}{2} \times \frac{1}{3} = \frac{5}{6} \end{array}\right\} = \frac{182}{24}.$$

$$\left.\begin{array}{l} 5\frac{1}{2} \times \frac{1}{2} = 2\frac{3}{4} \\ 2\frac{1}{2} \times \frac{1}{3} = \frac{5}{6} \end{array}\right\} = \frac{86}{24}.$$

$$2\frac{1}{2} \times \frac{1}{3} = \frac{5}{6} = \frac{20}{24}.$$

$$\text{Sum} = \frac{288}{24}.$$

A's share $= \frac{\frac{182}{24}}{\frac{288}{24}}$ of \$187.20 $=$ \$118.30.

B's " $= \frac{\frac{86}{24}}{\frac{288}{24}}$ of \$187.20 $=$ \$55.90.

C's " $= \frac{\frac{20}{24}}{\frac{288}{24}}$ of \$187.20 $=$ \$13.

Examples (cxiii) Page 225.

1. RESOURCES AND LIABILITIES.

DR.	CR.
\$3456	\$3250
2120	346
1874	
630	\$3596
8080	Resources at closing.
3596	Liabilities.
4484	Present worth of firm.
2510	Credit excess of Ownership.
1974	Net gain.
987	A's share of net gain.
987	B's " " "

OWNERSHIP.

DR.		CR.
\$175	A withdrew	\$1500
315	B "	1500
490	Total investment	3000
	" withdrawn	490
	Firm's net investment	2510

Hence A's net capital $= \$(1500 - 175 + 987) = \$2312$.

B's " " $= \$(1500 - 315 + 987) = \$2172$.

2. RESOURCES AND LIABILITIES.

DR.	CR.
$1424	$2450
1562	1240
383	
3485	3690
826	
7680	Resources at closing.
3690	Liabilities.
3990	Present worth of firm.

OWNERSHIP.

DR.	CR.
$1000	$6000
685	420
1860	4000
	280
3545	
	Investment 10700
	Withdrawal 3545
	Net investment 7155
	Debit excess of R. & L. 3990
	Net loss 3165
	A's $\frac{3}{5}$ of net loss 1899
	B's $\frac{2}{5}$ of " " 1266

Hence A's net capital at closing

$= \$\{(6000+420)-(1000+685+1899)\} = \$2836$;

and B's net capital at closing

$= \$\{(4000+280)-(1860+1266)\} = \$1154.$

3. RESOURCES AND LIABILITIES.

DR.	CR.
$2263	$3846
5000	4462
	675
7263	
Liabilities at closing	8983
Resources "	7263
Insolvency of firm	1720

OWNERSHIP.

DR.	CR.
$2860	$6000
5560	4000
	251
8420	
Investment	10251
Withdrawal	8420
Net investment	1830
Credit excess of R. & L.	1720
Net loss	3550
A's $\frac{3}{5}$ of net loss	2130
B's $\frac{2}{5}$ "	1420

Hence A's net capital at closing

$= \$(6000 - 2860 - 2130) = \$1010$;

and B's net capital at closing

$= \$(4250 - 5560 - 1420) = -\$2730,$

i. e., B's net insolvency = $2730.

Examples (cxiv). Page 230.

1. Diffs. 30 | 7 23 5 | ... | ... | 5 35 7

Arranging as in Ex. 2, we see that 5 lbs. at 23 cents, and 7 lbs. at 35 cents, will form a mixture that may be sold for 30 cents a pound.

2. Diffs. 55

20	35	3
...	...	...
5	60	2
25	80	2

We see that 3 bushels of oats, 2 bushels of rye, and 2 bushels of barley would form the required mixture. Of course, any multiples of these quantities would satisfy the conditions equally well, so that we might take 30 bushels of oats, 20 bushels of rye, and 20 bushels of barley.

3. Diffs. 70

15	55	2, 5
...	...	...
5	75	2, 3
20	90	1, 3

We find that 2 lbs. at 55 cents, 2 lbs. at 75 cents, and 1 lb. at 90 cents, may be sold without gain or loss. But there are 30 lbs. at 90 cents. Hence we must have 2×30 lbs. $= 60$ lbs. at 55 cents, and 2×30 lbs. $= 60$ lbs. at 75 cents. Or, we may take 5 lbs. at 55 cents, 3 lbs. at 75 cents, and 3 lbs. at 90 cents; we will then have 50 lbs. at 55 cents, and 30 lbs. at 75 cents.

4. Diffs. 1.20

30	1.50	4
...	...	...
1.20	0	1

We see that 4 gallons of alcohol at \$1.50 and 1 gallon of water will form a mixture that may be sold for \$1.20 a gallon. But there are 15×4, or 60 gallons of alcohol in the mixture. There must, therefore, be 15×1, or 15 gallons of water.

5. 12 gals. at 36 cents each $= 432$ cents.
8 " 56 " $= 448$ "
20 gals. 880 cents.

Hence cost of 1 gal. $= \frac{880}{20} = 44$ cents.

The question now is, how many gallons of Kerosene oil, worth 60 cents per gallon, must be mixed with 20 gallons of another kind worth 44 cents per gallon, so that the mixture may be sold for 50 cents a gallon.

As before, we have

Diff's.	50	
6	44	5
...	...	...
10	60	3

But there are 20 gallons, or 4 times 5 gallons at 44 cents. We must, therefore, have 4 times 3, or 12 gallons of Kerosene oil.

6. 16 bushels at 48 cents each = 768 cents.
12 " 34 " = 408 "

28 bushels 1176

Therefore the cost of 1 bushel = $\frac{1176}{28}$ cents
= 42 cents.

As in the previous question, we have

Diff's.	56	
14	42	2
...	...	...
4	60	1
24	80	1

That is, 2 bushels at 42 cents will balance 1 bushel at 60 cents and 1 bushel at 80 cents. But there are 28 bushels at 42 cents. There must, therefore, be 14 bushels of rye and the same quantity of barley.

7.

Diff's.	24	
10	14	3
6	18	3
...	...	
6	30	8

We find the proportional parts to form the mixture to be 3 lbs. at 14 cents, 3 lbs. at 18 cents, and 8 lbs. at 30 cents. Adding these proportional quantities we find that they form a mixture of 14 lbs. But the required mixture is to contain 84 lbs. Hence $\frac{84}{14} = 6$ = the number of times the proportional quantities must be increased in order to give the required quantity of the mixture. We shall, therefore, have

6×3 lbs. = 18 lbs. at 14 cents,
6×3 lbs. = 18 lbs. at 18 "
and 6×8 lbs. = 48 lbs. at 30 "

8. Diffs.	39		If we take the first proportional parts indicated, we have 1 lb. at 33 cents, 3 lbs. at 37 cents, and 2 lbs. at 45 cents. Adding, we find the proportional parts form a mixture of 6 lbs. But the required mixture must contain 120 lbs. Hence $\frac{120}{6} = 20 =$ the number of times the proportional parts must be increased in order to give the required quantity of the mixture. We shall, therefore, have 20×1 lb. $= 20$ lbs. at 33 cents, 20×3 lbs. $= 60$ lbs. at 37 cents, and 20×2 lbs. $= 40$ lbs. at 45 cents. If we take the second proportional parts, viz., 3, 3, and 4, we find that they form a mixture of 10 lbs. Hence $\frac{120}{10} = 12 =$ the number of times the proportional parts must be increased. Hence, we have
	—		
6	33	1, 3	
2	37	3, 3	
...	...	...	
6	45	2, 4	

12×3 lbs. $= 36$ lbs. at 33 cents, 12×3 lbs. $= 36$ lbs. at 37 cents, and 12×4 lbs. $= 48$ lbs. at 45 cents.

Examples (cxv). Page 237.

1. Since £1500 = \$7300;

$\therefore$ £1 $= \$4\frac{13}{15}$.

Now the advance on $\$4\frac{4}{9} = \$(4\frac{13}{15} - 4\frac{4}{9}) = \$\frac{19}{45}$;

$\therefore$ " " " \$100 $= \$\frac{100 \times \frac{19}{45}}{4\frac{4}{9}} = \$9\frac{1}{2}$.

Hence exchange is at a premium of $9\frac{1}{2}\%$ and the quotation would be $109\frac{1}{2}$.

2. Since 5·3 fr. = \$1;

$\therefore$ 236874 fr. $= \$\frac{236874 \times 1}{5.3}$

$= \$44693.20...$

3. Since 12 fl. = 25.56 fr.;

$\therefore$ 1 fl. $= \frac{25.56}{12}$ fr. $= 2.13$ fr.

$= 2$ fr. 13 cent.

4. Since $25\frac{1}{2}$ fr. $= 2244$ copecks;

20 fr. $= \frac{20 \times 2244}{25\frac{1}{2}}$ copecks

$= 1760$ copecks.

5. Since $25\frac{1}{2}$ fr. $= 11\frac{54}{60}$ fl.;

$\therefore$ 20 fr. $= \frac{20 \times 11\frac{54}{60}}{25\frac{1}{2}} = 9\frac{1}{3}$ fl.

$= 9$ fl. 20 kr.

6. Since $5.12\frac{1}{2}$ fr. $=$ \$1 (gold);

$\therefore$ 12669 fr. $=$ \$$\frac{12669 \times 1}{5.12\frac{1}{2}}$ (gold)

$=$ \$2472 (gold).

Now \$100 (gold) $=$ \$$135\frac{1}{3}$ (currency);

$\therefore$ \$2472 " $=$ \$$\frac{2472 \times 135\frac{1}{3}}{100}$ (currency)

$=$ \$3345.44.

7. Since $\frac{108}{100}$ of \$$4\frac{4}{9}$ $=$ £1;

$\therefore$ \$2767.80 $=$ £$\frac{2767.80 \times 1}{\frac{108}{100} \text{ of } 4\frac{4}{9}}$

$=$ £$\frac{2767.80 \times 100 \times 9}{108 \times 40}$

$=$ £576. 12*s*. 6*d*.

8. Amount of gold in \$1 $= \frac{9}{10}$ of $\frac{258}{10}$ gr.

$= 23.22$ gr.

Amount of gold in £1 $= \frac{11}{1\frac{1}{2}}$ of $\frac{40 \times 5760}{1869}$ gr.

$= \frac{11 \times 40 \times 480}{1869}$ gr.

Now, 23.22 gr. $=$ \$1;

$\therefore$ $\frac{11 \times 40 \times 480}{1869}$ gr. $=$ \$$\frac{11 \times 40 \times 480 \times 1}{23.22 \times 1869}$

$=$ \$4·8665....

9. Since 100 fl. $= 209{\cdot}25$ fr.;

$\therefore$ $12.16\frac{1}{4}$ fl. $= \frac{12.1625 \times 209.25}{100}$

$= 25$ fr. $45\ldots$ cents.

10. Since 18 fl. = 20 mar. ban.;
$\therefore$ 30 fr., or 14 fl. = $\frac{14\times20}{18}$ " ;
and £1, or 25.5 fr. = $\frac{25.5\times14\times20}{30\times18}$ mar. ban.
= $13\frac{2}{3}$ mar. ban.

11. Since 14 mar. ban. = £1;
$\therefore$ 20 mar. ban., or 18 fl. = £$\frac{20\times1}{14}$,
and 28 fl., or 60 fr = £$\frac{28\times20\times1}{18\times14}$,
4 fr., or 72 cents = £$\frac{4\times28\times\times20\times1}{60\times18\times14}$;
$\therefore$ \$1 = £$\frac{100\times4\times28\times20\times1}{72\times60\times18\times14}$
= £$\frac{100}{486}$.
Hence £1 = \$4.86.

12. Since 20 fr. = 40.5 fl.;
$\therefore$ £1, or 25.7 fr. = $\frac{25.7\times40.5}{20}$ fl.;
and £50 = $\frac{50\times25.7\times40.5}{20}$ fl.
= 2602.125 fl.

13. Since 25.65 fr. = 240 d.;
$\therefore$ 3 fr., or 525 rees = $\frac{3\times240}{25.65}$ $d.$
and 1 ree = $\frac{3\times240}{525\times25.65}d.$;
$\therefore$ 1000 rees = $\frac{1000\times3\times240}{525\times25.65}d.$
= $53\frac{1}{2}d.$ nearly.

14. Since 1 oz. Eng. gold = $\frac{3151}{3100}$ oz. of Fr. gold;
$\therefore$ 1 oz. English gold = $\frac{31.51}{3100}$ of 31.1 grammes.
Now 10 gram. = 31 fr.;
$\therefore$ $\frac{3151\times31.1}{3100}$ gram. = $\frac{3151\times31.1\times31}{10\times3100}$ fr.;
$\therefore$ 1 oz. Eng. gold = $\frac{3151\times31.1\times31}{10\times3100}$ fr. Fr. gold
and hence 1 fr. Fr. gold = $\frac{10\times3100}{3151\times31.1\times31}$ oz. Eng. gold
= ·0102045 oz. of Eng. gold.

Now $77\frac{1}{8}s. = 1$ oz.;

$\therefore \quad £1 = \frac{20 \times 1}{77\frac{1}{8}}$ oz.

And $\cdot 0102045$ oz. $= 1$ fr.;

$\therefore \frac{20}{77\frac{1}{8}}$ oz. $= \frac{20}{77\frac{1}{8} \times \cdot 0102045}$ fr.

$= 25.17$ fr.

EXAMINATION PAPERS.

I.—Page 238.

1.

$3 \times$	$.95 =$	2.85.
$7 \times$	$1.15 =$	8.05.
$12 \times$	$1.36 =$	16.32.
22		27.22

Hence sp. gr. of mixture $= \frac{27.22}{22}$

$= 1.2372...$

2. Sum of which \$6291 is the interest

$= \$\frac{6291 \times 100}{8 \times 4\frac{1}{2}}$

$= \$17475.$

Take C's money as the unit; then,

B's " $= \frac{3}{5}$ of $\frac{3}{5}$ of C's $= \frac{9}{10}$ of C's.

A's " $= \frac{3}{2}$ of $\frac{3}{4}$ of $\frac{9}{10}$ of C's

$= \frac{81}{80}$ of C's.

Sum of all their money $= C$'s $+ \frac{9}{10}$ of C's $+ \frac{81}{80}$ of C's

$= \frac{233}{80}$ of C's;

$\therefore \quad \frac{233}{80}$ of C's $= \$17475$;

and C's $= \$\frac{80 \times 17475}{233}$

$= \$6000.$

B's $= \frac{9}{10}$ of \$6000

$= \$5400.$

A's $= \frac{81}{80}$ of \$6000

$= \$6075.$

3. Since 21 fr. = \$4 ;

$\therefore$ 19 mar. ban., or 35 fr. = $\$\frac{35\times4}{21}$

and £7, or 96 mar. ban. = $\$\frac{96\times35\times4}{19\times21}$

and £1200 = $\$\frac{1200\times96\times35\times4}{7\times19\times21}$

= \$5774.43...

4. By direct exchange $\$100\frac{3}{4}$ = \$100 at New York ;

$\therefore$ \$14331.60 = $\$\frac{14331.60\times100}{100\frac{3}{4}}$

= \$14224.91....

\$1 Cinn. = $\$(\frac{1}{1.005})$ St. Louis

= $\$(\frac{1}{.995}\times\frac{1}{1.005})$ N. O.

= $\$(\frac{1}{.99}\times\frac{1}{.995}\times\frac{1}{1.005})$ N. Y. ;

$\therefore$ \$14331.60 Cinn. = $\$\frac{14331.60}{.99\times.995\times1.005}$ N. Y.

= \$14476.72....

Hence gain = \$251.81.

5. Cost of exchange of \$2660 = \$2570.89 ;

$\therefore$ " " \$1 = $\$\frac{2570.89}{2660}$

= \$·9665.

But the bank had the use of this money for 63 days, and allowed a deduction for interest.

Bank discount for 63 da. = $\$(\cdot9665\times\frac{63}{365}\times\frac{6}{100})$

= \$·010009.

Course of exchange = \$·976509...

$1-\cdot9765\ldots = 0234$;

$\therefore$ exchange was at a discount of 2·34 %.

II.—Page 238.

1. Cost of 1 lb. of the mixture = $\frac{87\times100}{116}$ cents

= 75 cents.

$\frac{1}{16}$ of the mixture consisted of the good tea and $\frac{5}{16}$ of it of the inferior kind.

Hence $\frac{11}{16}$ of cost of dear tea + $\frac{5}{16}$ of (cost of dear tea − 12 cents) = 75 cents ;

$\therefore$ cost of dear tea = $(75 + \frac{60}{16})$ cents ;

$= 78\frac{3}{4}$ cents,

and cost of cheap tea = $66\frac{3}{4}$ cents.

2. Sum expended in paying clerks = \$1600.

Sum given to A = $\$\frac{20000 \times 8}{100}$ = \$1600.

" " " B = $\$\frac{30000 \times 4}{100}$ = \$1200.

Sum to be apportioned = \$(12800 − 1600 − 1200 −1600−120) = \$8280.

Part of this given to A = $\frac{2}{5}$ of \$8280 = \$3312.

" " " " B = $\frac{3}{5}$ of \$8280 = \$4968.

Net sum received by A = \$(1600+3312) = \$4912.

" " " " B = \$(1200+4968) = \$6168.

3. Since $57\frac{1}{2}$ fl. = 120 fr.;

$\therefore$ £1, or 12.15 fl. = $\frac{12.15 \times 120}{57\frac{1}{2}}$ fr.

$= 25.35\frac{15}{23}$ fr.

4. $3 \times 1400 = 4200$
$2 \times 3400 = 6800$
$3 \times 1800 = 5400$
$4 \times 4200 = 16800$
$= 33200.$

$4 \times 2000 = 8000$
$6 \times 1400 = 8400$
$2 \times 4000 = 8000$
$= 24400.$

57600.

A's share = $\frac{32}{57}\frac{2}{60}\frac{0}{0}$ of \$4032 = \$2324.

B's " = $\frac{24}{57}\frac{4}{60}\frac{0}{0}$ of \$4032 = \$1708.

5. Every gal. of the first mixture contains $\frac{12}{30}$ or $\frac{2}{5}$ gal. of wine, and every gal. of the second mixture contains $\frac{9}{12}$ or $\frac{3}{4}$ gal. of wine ;

$\therefore$ $\frac{2}{5}$ of number from 1st + $\frac{3}{4}$ of (14 − number from 1st)
$= 7$ gal.;

$\therefore$ $(\frac{3}{4} - \frac{2}{5})$ of number from 1st $= (10\frac{1}{2} - 7)$ gal.,

and $\frac{7}{20}$ " " $= 3\frac{1}{2}$ gal.,

" " $= \frac{20 \times 3\frac{1}{2}}{7}$

$= 10.$

Hence the number from the 2nd $= 4.$

III.—Page 239.

1. By Art. 206 it is found that a mixture of 5 lb. at 8 cents, 5 lb. at 10 cents, 5 lb. at 12 cents, and 15 lb. at 20 cents, would be worth 15 cents per lb.

$5+5+5+15 = 30;$

$\therefore$ quantity at 8 cents $= \frac{5}{30}$ of 200 lb.
$= 33\frac{1}{3}$ lb.

" 10 cents $= \frac{5}{30}$ of 200 lb.
$= 33\frac{1}{3}$ lb.

" 12 cents $= \frac{5}{30}$ of 200 lb.
$= 33\frac{1}{3}$ lb.

" 20 cents $= \frac{15}{30}$ of 200 lb.
$= 100$ lb.

2. Interest for 18 da. at 6 % $= \frac{216}{73000}$ of note.

Discount $= \frac{3}{200}$ "

Note $- (\frac{216}{73000} + \frac{3}{200})$ of note $= \$1190{\cdot}234,$

and $\frac{71689}{73000}$ " $= \$1190{\cdot}234;$

$\therefore$ note $= \$\frac{73000 \times 1190{\cdot}234}{71689}$

$= \$1212,$ nearly.

3. A gain of \$120 in 6 mo. = a gain of \$20 in 1 mo.

" \$150 in 5 mo. = " \$30 "

" \$210 in 9 mo. = " \$23$\frac{1}{3}$ "

\$73$\frac{1}{3}$

Sum from which $\$23\frac{1}{3}$ is gained $= \$400$;

$\therefore$ " " $\$73\frac{1}{3}$ " $= \$\frac{73\frac{1}{3}\times 400}{23\frac{1}{3}}$

$= \$1257\frac{1}{7}$.

4. After each drawing off $\frac{3}{4}$ of the wine remaining in the cask is left.

Hence the part finally left

$= \frac{3}{4}$ of $\frac{3}{4}$ of $\frac{3}{4}$ of $\frac{3}{4}$ of the wine

$= \frac{81}{256}$ of the wine.

5. Since 25.15 fr. $= £1$;

$\therefore$ 1 rouble, or 1.2 fr. $= £\frac{1.2\times 1}{25.15}$,

and 920 roubles $= £\frac{920\times 1.2\times 1}{25.15}$

$= £43\ 17s.\ 11d.$ (nearly).

Again, since 25.35 fr. $= £1$;

$\therefore$ 1 rouble, or 1.15 fr. $= £\frac{1.15\times 1}{25.35}$,

and 920 roubles $= £\frac{920\times 1.15\times 1}{25.35}$

$= £41\ 14s.\ 8\frac{1}{2}d.$ (nearly)

Hence the broker's gain $= £2\ 3s.\ 2\frac{1}{2}d.$

IV.—Page 240.

1. $£354\ 16s.\ 3d. = 85155d.$

Since $38\frac{1}{2}d. = 1$ dollar;

$\therefore 85155d. = \frac{85155\times 1}{38\frac{1}{2}}$ dollars

$= 2211\frac{3}{11}$ dollars.

2. Since 1 lira $= \$0.22$;

$\therefore$ 7500 lire $= 7500\times \$0.22$

$= \$1650.$

By circuitous exchange $£1 = \$4.95$;

$\therefore$ 26 fr. $= \$4.95$,

and 1 lira, or $1\frac{1}{8}$ fr. $= \$\frac{1\frac{1}{8}\times 4.95}{26}$;

$\therefore$ 7500 lire $= \$\frac{7500\times 9\times 4.95}{8\times 26}$

$= \$1606.37\ldots$

Hence the difference $= \$43.63.$

3. Since £200 = $1000;
$\therefore$ £1 = $5.

4. By direct exchange £1 = $4.86\frac{2}{3}$;
$\therefore$ £3000 = $3000 \times \$4.86\frac{2}{3}$
= $14600.

Through Paris 5.25 fr. = $1;
$\therefore$ £1, or 25 fr. = $\$\frac{25 \times 1}{5.25}$,
and £3000 = $\$\frac{3000 \times 25 \times 1}{5.25}$
= $14285\frac{5}{7}$.

Through Amsterdam 1 guild. = $0.40;
$\therefore$ £1, or $12\frac{1}{5}$ guild. = $12\frac{1}{5} \times \$0.40$,
and £3000 = $3000 \times 12\frac{1}{5} \times \$0.40$
= $14640.

By direct exchange he has to pay $14600 for the draft; by Paris, only $14285\frac{5}{7}$, and by Amsterdam $14640.

5. Cost price $\frac{100}{116}$ of $14\frac{1}{2}$ cents = $12\frac{1}{2}$ cents.

Diff.	$12\frac{1}{2}$			
$4\frac{1}{2}$	1 lb. at	8	1	
$9\frac{3}{4}$	3 "	$9\frac{1}{4}$	1	= $22\frac{1}{4}$ gain.
8	2 "	$8\frac{1}{2}$	1	
...	.....		...	
$\frac{1}{2}$	1 "	13	$8\frac{1}{2}$ = $4\frac{1}{4}$	
$1\frac{1}{2}$	1 "	14	8 = 12	= $22\frac{1}{4}$ loss.
6	4 "	14	1 = 6	

We have, therefore, 1 lb. at 8 cents,
$8\frac{1}{2}$ lb. " 13 "
and 8 lb. " 14 " .

Of course, the above are only a few of the many answers that might be found to this question.

V.—Page 240.

1. 2456 + 735 + 4361 = 7552.

Number to be provided by 1st $= \frac{2456}{7532}$ of 182
$= 59$, nearly.
" " " 2nd $= \frac{735}{7532}$ of 182
$= 17$, nearly.
by 3rd $= \frac{4361}{7532}$ of 182
$= 106$, nearly.

2. Cost of 9 gal. of mixture $= 70s.$
$\therefore$ " 1 gal. " $= \frac{70}{9}s. = 7\frac{7}{9}s.$
Selling price of 1 gal. " $= 6 \times 2\frac{5}{6}s. = 17s.$
Gain on $7\frac{7}{9}s. = 9\frac{2}{9}s.$;
$\therefore$ " $100s. = \frac{100 \times 9\frac{2}{9}}{7\frac{7}{9}}$
$= 118\frac{4}{7}s.$

3. Since the gain on \$2200 $=$ \$880;
$\therefore$ " " \$3500 $= \$\frac{3500 \times 880}{2200}$
$=$ \$1400.
But the gain for 2 mo. less $=$ \$1120;
$\therefore$ 2 mo. gain on \$3500 $=$ \$280.
Since time for which \$280 is gain on \$3500 $=$ 2 mo.;
$\therefore$ " " \$1120 " \$3500
$= \frac{1120 \times 2}{280}$
$= 8$ mo.
Time for which \$280 is gain on \$3500 $=$ 2 mo.;
$\therefore$ " " \$880 " \$2200
$= \frac{880 \times 3500 \times 2}{2200 \times 280}$
$= 10$ mo.
Time for which \$280 is gain on \$3500 $=$ 2 mo.:
$\therefore$ " " \$1200 " \$2500
$= \frac{1200 \times 3500 \times 2}{2500 \times 280}$
$= 12$ mo.

4. Capital at end of 1st year
$= \frac{3}{2}$ of original capital $-$ £1200.
Capital at end of 2nd year
$= \frac{3}{2}$ of ($\frac{3}{2}$ of original capital $-$ £1200) $-$ £1200
$= \frac{9}{4}$ of original capital $-$ £3000.
Capital at end of 3rd year
$= \frac{3}{2}$ of ($\frac{9}{4}$ of original capital $-$ £3000) $-$ £1200
$= \frac{27}{8}$ of original capital $-$ £5700.
Capital at end of 4th year
$= \frac{3}{2}$ of ($\frac{27}{8}$ of original capital $-$ £5700) $-$ £1200
$= \frac{81}{16}$ of orignal capital $-$ £9750.
Hence $\frac{81}{16}$ of original capital $-$ £9750 $= 4 \times$ original capital;

$$\therefore \ \tfrac{17}{16} \text{ of original capital} = £9750,$$
$$\text{and original capital} = £\tfrac{16 \times 9750}{17}$$
$$= £9176\tfrac{8}{17}$$

5. Strength of 1 gal. of the mixture
$= (\frac{34}{100} + \frac{2 \times 46}{100}) \div 3 = \frac{42}{100}$.
Since the gain on $\frac{34}{100} = \frac{8}{100}$;

$$\therefore \quad \text{“} \quad \text{“} \quad 100 = \frac{100 \times \frac{8}{100}}{\frac{34}{100}}$$
$$= 23\tfrac{9}{17}.$$

Examples (cxvi). Page 242.

9. Relation $36 \times 640 : 180 = \frac{36 \times 640}{180} : 1$
$= 128 : 1.$

10.
$$\begin{array}{r} 7 : 8 \\ 12 : 15 \\ \frac{1}{2} : \frac{1}{5} \\ \hline 7 : 4 = \text{compound ratio.} \end{array}$$

Hence the 4th ratio $= \frac{63}{7} : \frac{52}{4}$
$= 9 : 13.$

11. If $\frac{3}{5}$ be the given ratio. Then adding any number, say 5, to each of the terms, we have $\frac{2+5}{3+5} = \frac{7}{8}$. Comparing this ratio with $\frac{3}{5}$, we have $\frac{16}{24}$ and $\frac{21}{24}$. Hence we see that this ratio is increased by adding the same number to each of its terms.

Again, if we take $\frac{4}{5}$ as the ratio and add, say 2, to each of the terms, we have $\frac{4+2}{3+2} = \frac{6}{5}$. Comparing this with the original ratio, we have $\frac{20}{15}$ and $\frac{18}{15}$. Hence, we see that this ratio is diminished by adding the same number to each of its terms. A ratio is, therefore, increased or diminished by adding the same number to each of its terms according as the antecedent is less or greater than its consequent.

Examples (cxvii.) Page 245.

4. $B = \frac{5}{26}$ of C;
$A = \frac{10}{3}$ of $\frac{5}{26}$ of C
$= \frac{25}{39}$ of C;
$\therefore A : C :: 25 : 39.$

10. B's share $= \frac{5}{6}$ of A's.
C's " $= \frac{3}{4}$ of B's $= \frac{3}{4}$ of $\frac{5}{6}$ of A's.
D's " $= \frac{2}{3}$ of C's $= \frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of A's;
$\therefore A$'s $+ \frac{5}{6}$ of A's $+ \frac{5}{8}$ of A's $+ \frac{5}{12}$ of A's $= \$1587$;
$\therefore (1 + \frac{5}{6} + \frac{5}{8} + \frac{5}{12})$ of A's $= \$1587$,
and $\frac{69}{24}$ of A's $= \$1587$.

A's $= \$\frac{24 \times 1587}{69} = \$552.$
B's $= \frac{5}{6}$ of $\$552 = \$460.$
C's $= \frac{3}{4}$ of $\$460 = \$345.$
D's $= \frac{2}{3}$ of $\$345 = \$230.$

Examples (cxviii.) Page 247.

1. $233d.$: £1247 $10s.$ $5d.$ :: £1 : gross income ;

$$\therefore \text{ gross income} = £\frac{299405 \times 1}{233}$$
$$= £1285.$$

2. The number of hours between 12 at noon on Monday and $10\frac{1}{4}$ a.m. on Saturday $= 118\frac{1}{4}$ hr.

24 hr. : $118\frac{1}{4}$ hr. :: 3 m. 10 s. : time gained ;

$$\therefore \text{ time gained} = \frac{118\frac{1}{4} \times 3\frac{1}{6}}{24} \text{ min.}$$
$$= 15 \text{ min. } 36\tfrac{7}{48} \text{ s.}$$

As the watch was 10 min. fast on Monday, it is now 25 m. and $36\frac{7}{48}$ s. too fast, and hence it is 10 h. 40 m. $36\frac{7}{48}$ s.

3. Gain in $6\frac{1}{2}$ rounds $= \frac{1}{3}$ mi.

$6\frac{1}{2}$ rounds : 9 rounds :: $\frac{1}{3}$ mi. : A's gain. ;

$$\therefore A\text{'s gain} = \frac{9 \times \frac{1}{3}}{6\frac{1}{2}} \text{ mi.}$$
$$= \tfrac{6}{13} \text{ mi.}$$

4. The hands of the watch will be together for the 11th time after noon at $16\frac{4}{11}$ min. past 3. Art. 173.

The watch will have gone $(6\frac{7}{11} + 180 + 16\frac{4}{11})$ min., or 203 min.

But $59\frac{17}{41}$ min. on the watch correspond to 60 min. of true time ;

$\therefore$ $59\frac{17}{41}$: 203 :: 60 min. : time required ;

$$\therefore \text{ time required} = \frac{203 \times 60}{59\frac{17}{41}} \text{ min.} = 205 \text{ min.}$$
$$= 3 \text{ hr. } 25 \text{ min.}$$

5. Since 4 men $=$ 6 women $=$ 9 boys ;

$\therefore$ 1 man $= \frac{6}{4}$ " $= \frac{9}{4}$ " ;

and 5 men $= \frac{30}{4}$ " $= \frac{45}{4}$ " .

$(9+\frac{3}{4}^0)$ women : 6 women :: $27\frac{1}{2}$ da. : time required ;

$$\therefore \text{ time required } = \frac{27\frac{1}{2}\times 6}{9+\frac{3}{4}^0} \text{ da.}$$

$$= 10 \text{ da.}$$

$(8+\frac{4}{4}^5)$ boys : 9 boys :: $27\frac{1}{2}$ da. : time required ;

$$\therefore \text{ time required } = \frac{27\frac{1}{2}\times 9}{8+\frac{4}{4}^5} \text{ da.}$$

$$= 12\tfrac{6}{7} \text{ da.}$$

6. $14\frac{3}{8} : 5\frac{5}{6}$ :: \$116.15 : value required ;

$$\therefore \text{ value required } = \$\frac{5\frac{5}{6}\times 116.15}{14\frac{3}{8}}$$

$$= \$47.13\tfrac{1}{3}.$$

7. 26 in. : (7×9) in. :: $32\frac{1}{2}$ yds. : yards required ;

$$\therefore \text{ yards required } = \frac{7\times 9\times 32\frac{1}{2}}{26}$$

$$= 78\tfrac{3}{4}.$$

8. The difference in 24 hr. $= 7\frac{1}{2}$ min.

Their present difference $= 5$ min.

In how many hours will their difference amount to 25 min. ?

$7\frac{1}{2}$ min. : 25 min. :: 24 hr. : hr. required ;

$$\therefore \text{ hours required } = \frac{25\times 24}{7\frac{1}{2}}$$

$$= 80.$$

80 hours from noon on Monday is 8 p.m. on Thursday.

9. $2\frac{2}{3} : 3\frac{1}{4}$ :: 6336 stones : stones required ;

$$\therefore \text{ stones required } = \frac{3\frac{1}{4}\times 6336}{2\frac{2}{3}}$$

$$= 7722.$$

10. 7 : 3 :: 22400 people : number fed ;

$$\therefore \text{ number fed } = \frac{22400\times 3}{7}$$

$$= 9600 ;$$

$$\therefore \text{ number sent away } = 22400 - 9600$$

$$= 12800.$$

Examples (cxix). Page 249.

1. $60 : 12 :: 18$ men : men required;
$40 : 360$
$3 : 8$
$16 : 10$
$\therefore$ men required $= \frac{18 \times 12 \times 360 \times 8 \times 10}{60 \times 40 \times 3 \times 16}$
$= 54.$

2. In 18 months 1200 men complete $\frac{3}{5}$ of the work; how many men will be required to do $\frac{5}{8}$ of the work in 16 months.

$16 : 18 :: 1200$ men : number required;
$\frac{3}{5} : \frac{5}{8}$
$\therefore$ number required $= \frac{1200 \times 18 \times \frac{5}{8}}{16 \times \frac{3}{5}}$
$= 2250;$
$\therefore$ number additional $= 2250 - 1200$
$= 1050.$

3. $6 : 7 :: 9$ men : number required;
$5 : 6$
$7 : 10$
$\therefore$ number required $= \frac{9 \times 7 \times 6 \times 10}{6 \times 5 \times 7}$
$= 18.$

4. $185 : 92\frac{1}{2} :: 20$ men : number required;
$1\frac{4}{5} : 9$
$\therefore$ number required $= \frac{20 \times 92\frac{1}{2} \times 9}{185 \times 1\frac{4}{5}}$
$= 50.$

5. 4 times work of soldiers + 4 times work of navvies = work necessary to dig the trench in 1 day;
And 7 times work of soldiers + 7 times work of half the navvies = work necessary to dig the trench in 1 day;

$\therefore$ 8 times work of soldiers + 8 times work of navvies = 14 times work of soldiers + 7 times work of navvies; and hence work of navvies = 6 times work of soldiers.

6. $2 : 10 :: 1$ drona : amount required;
$10 : 12\frac{1}{2}$
$9 : 11\frac{1}{4}$
$36 : 45$
$7 : 8\frac{3}{4}$
$\therefore$ amount required $= \frac{10 \times 12\frac{1}{2} \times 11\frac{1}{4} \times 45 \times 8\frac{3}{4}}{2 \times 10 \times 9 \times 36 \times 7}$
$= 12\frac{53}{256}$.

7. $470 : 360 :: 658$ revolutions : number required;
$7 : 8$
$\therefore$ number required $= \frac{658 \times 360 \times 8}{470 \times 7}$
$= 576$.

8. If 15 men working 15 hours a day do $\frac{3}{5}$ of a piece of work in 24 days, how many hours a day must 18 men work to do the rest of it in 12 days?
$18 : 15 :: 15$ hr. : hours required;
$12 : 24$
$\frac{3}{5} : \frac{2}{5}$
$\therefore$ hours required $= \frac{15 \times 15 \times 24 \times \frac{2}{5}}{18 \times 12 \times \frac{3}{5}}$
$= 16\frac{2}{3}$.

9. $24 : 248 :: 5\frac{1}{2}$ da. : days required;
$9 : 12$
$7 : 4$
$232\frac{1}{2} : 387\frac{1}{2}$
$3\frac{2}{3} : 5\frac{1}{4}$
$2\frac{1}{3} : 3\frac{1}{2}$
$\therefore$ days required $= \frac{5\frac{1}{4} \times 248 \times 12 \times 4 \times 387\frac{1}{2} \times 5\frac{1}{4} \times 3\frac{1}{2}}{24 \times 9 \times 7 \times 232\frac{1}{2} \times 3\frac{1}{3} \times 2\frac{1}{3}}$
$= 155$.

10. 9 : 5 :: 16 da. : days required;
10 : 11
25 : 36
24 : 16
44 : 50
40 : 45
$\therefore$ days required $= \frac{16\times5\times11\times36\times16\times50\times45}{9\times10\times25\times24\times44\times40}$
$= 12.$

Examples (cxxiv). Page 258.

1. Area of floor $= (14\frac{1}{3}\times15\frac{1}{2})$ sq. ft.
$= \frac{43\times31}{9\times6}$ sq. yd.;
$\therefore$ cost $= \frac{43\times31\times20}{9\times6}$ cents
$= \$4.93\frac{1}{27}$.

2. Area $= (146\frac{3}{4}\times88\frac{3}{4})$ sq. ft. $= \frac{587\times355}{9\times16}$ sq. yd.;
$\therefore$ cost $= \frac{587\times355\times36}{9\times16}$ cents $= \$520.96\frac{1}{4}$.

3. Since 4 ro. 1 po. 29 yd. $6\frac{3}{4}$ ft. $= 44100$ sq. ft.;
$\therefore$ side $= \sqrt{44100}$ ft. $= 210$ ft.

4. Since 1 ro. 26 po. 28 yd. $4\frac{1}{2}$ ft. $= 18225$ sq. ft.;
$\therefore$ side $= \sqrt{18225}$ ft. $= 135$ ft.

5. Area $= (40\times3\times100)$ sq. ft.
Number of turfs $= \frac{40\times3\times100}{3\times1}$;
$\therefore$ cost $= \frac{40\times3\times100\times81}{100\times3\times1}$ d.
$= £13\ 10\ s.$

6. Length of room $= \sqrt{289}$ ft. $= 17$ ft.
Area of walls $= (4\times17\times11\frac{1}{2})$ sq. ft.
$= 782$ sq. ft.
Area to be whitewashed $= (32\frac{1}{9}+\frac{782}{9})$ sq. yd.
$= 119$ sq. yd.;
$\therefore$ cost $= 119\times5$ cents $= \$5.95$.

7. Area of room $= (8\frac{1}{4} \times 6\frac{2}{3})$ sq. yd. $= 55$ sq. yd.
Length of carpet $= \frac{55}{\frac{2}{3}}$ yd. $= \frac{165}{2}$ yd.;
$\therefore$ cost per yard $= \$\frac{99}{\frac{165}{2}} = \$1.20.$

8. Since £2 19*s.* 8*d.* $= 716d.$;
$\therefore$ length of paper $= \frac{716}{4}$ yd.
Area of paper $= \left(\frac{716}{4} \times \frac{2}{3}\right)$ sq. yd.
Height of room $= \left\{\left(\frac{716}{4} \times \frac{2}{3}\right) \div (16\frac{1}{2} + 13\frac{1}{3})\right\}$ yd.
$= 12$ ft.

9. Area $= 559504$ sq. ft.
Breadth $= \frac{559504}{2992}$ ft. $= 187$ ft.

10. Area $= (330 \times 330)$ sq. yd.
$= 22\frac{1}{2}$ a.

11. Breadth $= \frac{\frac{5}{11} \text{ of the area of the walls}}{21}$
$= \frac{\frac{5}{11}\{(42 + 2 \times \text{breadth}) \times 10\frac{1}{2}\}}{21}$
$= \frac{5}{11}\{21 + \text{breath}\}$
$= \frac{105}{11}$ ft. $+ \frac{5}{11}$ of breadth;
$\therefore \frac{6}{11}$ of breadth $= \frac{105}{11}$ ft.;
$\therefore$ breadth $= \frac{11}{6}$ of $\frac{105}{11}$ ft.
$= 17\frac{1}{2}$ ft.

12. Length $= \frac{144}{6\frac{3}{4}}$ in. $= 1$ ft. $9\frac{1}{3}$ in.

13. Area of room
$= \left\{2 \times (12\frac{9}{16} + 9\frac{15}{16}) \times 10\right\}$ sq. ft.
$= (2 \times 22\frac{1}{2} \times 10)$ sq. ft.
Yards of paper $= \frac{2 \times 22\frac{1}{2} \times 10}{3 \times \frac{3}{2}}$
$= 100.$

And 100 yds., at 12 cents a yard = $12.

14. Number $= \frac{124 \times 124}{2 \times \frac{31}{24}}$
$= 5952.$

15. Area of walls
$= \{ (2 \times 15 + 2 \times 12) \times 10 \}$ sq. ft.
$= 540$ sq. ft.

Length of paper $= \frac{540}{\frac{5}{2}}$ ft. $= 216$ ft.

Cost of paper $= \frac{216}{3} \times 12\frac{1}{2}$ cents.
$=$ $9.

16. Area of walls
$= \{ (2 \times 21 + 2 \times 15) \times 12 \}$ sq. ft.
$= 864$ sq. ft.
Deduction $= \{21 + 30 + 2 \times 69\}$ sq. ft. $= 189$ sq. ft.

Length of paper $= \frac{675}{24}$ ft. $= 90$ yd.
Cost of paper $= 90 \times 15$ cents.
$=$ $13.50.

17. Since 50 a. 2 r. 32 po. $= 245388$ sq. yd.;
$\therefore$ breadth $= \frac{245388}{572}$ yd.
$= 429$ yd.
Diagonal $= \sqrt{(572^2 + 429^2)}$ yd.
$= \sqrt{511225}$ yd. $= 715$ yd.

18. Area of each part $= (\frac{900 - 20}{2} \times \frac{450 - 20}{2})$ sq. ft.
$= 10511\frac{1}{9}$ sq. yd.
Area covered by trees $= (900 \times 20 + 430 \times 20)$ sq. ft.
$= 2955\frac{5}{9}$ sq. yd.

19. Area of walls
$= \{(2 \times \text{length} + 2 \times \text{breadth}) \times 11\}$ sq. ft.
$= \{(4 \times \text{breadth} + 2 \times \text{breadth}) \times 11\}$ sq. ft.
$= (66 \times \text{breadth})$ sq. ft.;
$\therefore 66 \times \text{breadth} = (143 \times 3) \times 2,$
and breadth $= \frac{143 \times 6}{66}$ ft.
$= 13$ ft.,
and length $= 26$ ft.;
$\therefore$ length of moulding $= \frac{2 \times 26 + 2 \times 13}{3}$ ft.
$= 25$ yds.

20. Area of ceiling $= (27\frac{1}{3} \times 20)$ sq. ft.
$= \frac{1640}{3}$ sq. ft.
Area of walls $= \{(2 \times 27\frac{1}{3} + 2 \times 20) \times 12\frac{1}{2}\}$ sq. ft.
$= \frac{3550}{3}$ sq. ft.;
$\therefore$ area to be painted $= \frac{5190}{3}$ sq. ft.;
$\therefore$ cost $= \frac{5190}{9 \times 3} \times 36$ cents.
$= \$69.20$

21. Area of room $= (15\frac{3}{4} \times 13\frac{1}{3})$ sq. ft.
Length of carpet $= \frac{15\frac{3}{4} \times 13\frac{1}{3}}{2\frac{1}{4}}$ ft.;
$\therefore$ cost $= \frac{15\frac{3}{4} \times 13\frac{1}{3}}{3 \times 2\frac{1}{4}} \times 95$ cents.
$= \$29.55\frac{5}{9}$.

22. Area of room $= (10\frac{2}{3} \times 7\frac{1}{3})$ sq. yd.
Length of carpet $= \frac{10\frac{2}{3} \times 7\frac{1}{3}}{\frac{2}{3}}$ yd.
$= \frac{32 \times 22 \times 4}{3 \times 3 \times 3}$ yd.;
$\therefore$ cost $= \frac{32 \times 22 \times 4}{3 \times 3 \times 3} \times \$1.08$
$= \$112.64$.

23. Area of room $= (11 \times 8)$ sq. yd.
Length of carpet $= \frac{396}{3}$ yd.
$= 132$ yd.;

$\therefore$ width of carpet $= \frac{11 \times 8}{132}$ yd.
$= \frac{2}{3}$ yd.

24. Since 12.45 ft. $=$ 4.15 yd.;
$\therefore$ length to be paved
$= \{(2 \times 45.77 + 2 \times 4.15 + 2 \times 41.93)$yd.
$= 192$ yd.
Area to be paved $= (192 \times 4.15)$ sq. yd.;
$\therefore$ number of stones $= \frac{192 \times 4.15 \times 9}{5.76 \times 4.15}$
$= 300.$

Examples (cxxv.) Page 262.

6. Content of the wall $= (75 \times 12 \times 6 \times 12 \times 18)$ c. in.
Content of one brick $= (9 \times \frac{9}{2} \times 3)$ c. in.;
$\therefore$ number of bricks $= \frac{75 \times 12 \times 6 \times 12 \times 18}{9 \times \frac{9}{2} \times 3}$
$= 9600.$

7. Number of c.ft. of ice $= 45 \times 4840 \times 9 \times \frac{1}{4}$.
Weight in lbs. $= 45 \times 4840 \times 9 \times \frac{1}{4} \times \frac{920}{16}$
$= 14088\frac{15}{16}$ tons.

8. Number of men required to dig $(800 \times 500 \times 40)$ c. yd.
in 1 month $= 4 \times 500$;
$\therefore$ number of men required to dig $(1000 \times 400 \times 50)$ c. yd.
in 1 month $= \frac{1000 \times 400 \times 50 \times 4 \times 500}{800 \times 500 \times 40}$
$= 2500$;
hence number of men to do it in 5 mo. $= \frac{2500}{5}$
$= 500.$

9. Area of side $= \frac{8664}{24}$ sq in. $= 361$ sq. in.
Length of side $= \sqrt{361}$ in. $= 19$ in.

10. Content of cistern $= (4 \times 2\frac{1}{2} \times 3\frac{1}{4})$ c. ft.
Weight of water $= \frac{4 \times 5 \times 13 \times 1000}{16 \times 2 \times 4}$ lb.
$= 2031\frac{1}{4}$ lb.

11. Content of stone $= (4 \times 12 \times 30 \times 15)$ c. in.
Weight of $(4 \times 12 \times 30 \times 15)$ c. in. $= 27$ cwt.;
$\therefore$ weight of 100 c. in. $= \frac{100 \times 27}{4 \times 12 \times 30 \times 15}$ cwt.
$= \frac{1}{8}$ cwt.

12. 4 t. 12 cwt. 3 qr. 10 lb. 7 oz. $= 166375$ oz.;
$\therefore$ content of vessel $= \frac{166375}{1000}$ c. ft.
$= 166{\cdot}375$ c. ft.
Length of side $= \sqrt[3]{166{\cdot}375}$ ft.
$= 5{\cdot}5$ ft.

13. Number of men required to dig
$(\frac{3}{4} \times 1760 \times 30 \times 7)$ c. yd. in 1 day $= 42 \times 120$;
$\therefore$ number of men required to dig
$(1000 \times 36 \times \frac{2}{3})$ c. yd. in 1 day $= \frac{1000 \times 36 \times 22 \times 42 \times 120}{4 \times 1760 \times 30 \times 7 \times 3}$
$= 4800$;
hence number required to dig it in 30 days
$= \frac{4800}{30} = 160.$

14. Cubic content of cistern holding 2520 lb.
$= (9 \times \frac{16}{3} \times \frac{2}{4})$ c. ft.;
$\therefore$ " " " $= 3850$ lb.
$= \frac{3850 \times 9 \times 16 \times 9}{2520 \times 3 \times 4}$ c. ft.
$= 165$ c. ft.;
hence depth of cistern $= \frac{165}{8 \times 5\frac{1}{2}}$ ft.
$= 3\frac{3}{4}$ ft.

15. Cost of excavation $= (110 \times 6 \times \frac{1}{3})s.$ $= 220s.$
" rubble $= (110 \times 6 \times \frac{2}{9})s.$ $= 146\frac{2}{3}s.$
" gravel $= (110 \times 6 \times \frac{1}{4}) \times \frac{5}{2}s.$ $= 412\frac{1}{2}s$
Total cost $= 779\frac{1}{6}s$
$= £38\ 19s.\ 2d.$

EXAMINATION PAPERS.

Page 263.

1. Number $= \frac{40200-37601}{23} = \frac{2599}{23} = 113.$

2. Profit on 1 yard $= \$3.35 - \$3.20 = 15$ cents.
" " 500 yards $= 500 \times 15$ cents $= \$75.$

3. $372 = 2\times2\times3\times31$, or, 372, 837, 248.
$837 = 3\times3\times3\times31$, 124, 93, 248.
$248 = 2\times2\times2\times31$. 31, 93 62
31 is, therefore, the

H. C. F. (See pages 5 and 6.)

Since $\frac{7}{9} = \frac{13\times29\times7}{13\times29\times9} = \frac{2639}{13\times29\times9}$;
$\frac{11}{13} = \frac{9\times29\times11}{9\times29\times13} = \frac{2871}{13\times29\times9}$;
$\frac{24}{29} = \frac{9\times13\times24}{9\times13\times29} = \frac{2808}{9\times13\times29}$;
$\therefore$ the decreasing order is $\frac{11}{13}, \frac{24}{29}, \frac{7}{9}$.

4. Length of pace in inches $= \frac{33\times5280\times12}{7920}$
$= 30.$

5. Number of each $= 9366 \div (960+480+120+1)$
$= 6.$

6. $\frac{\cdot14}{7} = \cdot02.$
$\frac{140}{\cdot07} = \frac{14000}{7} = 2000.$
$\frac{\cdot014}{7000} = \cdot000002.$
Sum $= 2000\cdot020002$
$= \frac{2000020002}{1000000}$
$= \frac{1000010001}{500000}.$

7. $7.\dot{5}\dot{7} \times .3\dot{6} = 7\frac{57}{99} \times \frac{33}{90}$

$= \frac{25}{9}$.

$2.3\dot{4}\dot{5} = \frac{2322}{990}$.

$\frac{25}{9} - \frac{2322}{990} = \frac{2750-2322}{990}$

$= \frac{428}{990} = .4\dot{3}\dot{2}$.

8.

3	41707796 ft.
$5\frac{1}{2}$	13902598 yds. 2 ft.
40	2527745 po. $\frac{1}{2}$ yd.
8	63193 fur. 25 po.
	7899 mi. 1 fur.

7899 mi. 1 fur. 25 po. 3 ft. 6 in.

9. Distance passed over in 1 sec. $= \frac{66}{3}$ yd. $= 22$ yd.

" " 1 hr. $= \frac{60\times60\times22}{1760}$ mi.

$= 45$ mi.

10. Sum earned in 12 mos. $= 2 \times \$420 = \$840$.

Sum spent " $= 3 \times \$210 = \$630$.

Sum laid by " $= \$840 - \$630 = \$210$.

11. Number of steps $= \frac{4\frac{1}{2}\times5280\times12}{32}$

$= 9405$.

12. One gets 4 parts when the other gets 3.

$4 + 3 = 7$.

One gets $\frac{4}{7}$ of $\$13230 = \$7560$.

The other gets $\frac{3}{7}$ of $\$13230 = \$5670$.

13. $\frac{41}{162} - \frac{9}{49} - \frac{3}{54} = \frac{16}{81} - \frac{0}{49} = \frac{55}{3969}$.

$\frac{4}{9} + \frac{1}{2} - \frac{3}{14} = \frac{4}{9} + \frac{4}{14} = \frac{46}{63}$.

$\frac{55}{3969} \div \frac{46}{63} = \frac{55\times63}{3969\times46} = \frac{55}{2898}$.

$\frac{55}{2898} = .0189\ldots\ldots$

14. $\frac{(\frac{10}{3} - \frac{5}{2}) \div \frac{5}{16}}{\frac{8}{3} \div \frac{3}{4}} = \frac{5}{6} \times \frac{16}{5} \times \frac{3}{8} \times \frac{3}{4}$

$= \frac{3}{4} = .75$.

15. $\frac{4}{3} \times (\frac{22}{9} + \frac{68}{9}) = \frac{4}{3} \times \frac{90}{9} = \frac{40}{3}$.

$\frac{2341}{990} - \frac{1681}{990} = \frac{2}{3}$.

$\frac{40}{3} + \frac{2}{3} = \frac{42}{3} = 14$.

16. 11 ro. 11 po. 11 yd. $= 451$ ro. 11 yd. $= 13653\frac{3}{4}$ yd.

$= 122883\frac{3}{4}$ ft. $= 17695260$ in.

Fraction $= \frac{136531}{3 \times 4840} = \frac{54615}{3 \times 4840 \times 4} = \frac{331}{352}$.

17. Wages for 75 days $= 75 \times \$1.25 = \$93.75$.

Sum lost by not working $= \$93.75 - \$69.15 = \$24.60$.

Sum lost by not working 1 day

$= \$1.25 + \$.80 = \$2.05$.

Number of days he was idle $= \frac{24.60}{2.05} = 12$.

18. Number of men required in 1 hr. $= 10 \times 12 \times 24$.

" " " 80 hr. $= \frac{10 \times 12 \times 24}{80}$

$= 36$;

$\therefore$ " " " " to do 3 times as much $= 3 \times 36 = 108$.

19. Value of $\frac{3}{10}$ of estate $= \$7500$;

$\therefore$ " the estate $= \$\frac{10 \times 7500}{3}$;

$\therefore$ " $\frac{48}{100}$ of estate $= \$\frac{48 \times 10 \times 7500}{100 \times 3}$

$= \$12000$.

20. Whole sum remaining $= \$105.03$;

$\therefore$ sum each ought to have $= \$\frac{105.03}{3} = \$35.01$;

$\therefore$ *A* must hand over to *C* $\$37\ 50 - \$35.01 = \$2.49$;

$\therefore$ *B* " " $\$50.82 - \$35.01 = \$15.81$.

21. $\frac{1\frac{1}{4} - 1\frac{5}{12}}{1\frac{1}{4} + \frac{5}{12}} + \frac{7 \times 9 \times 5}{6 \times 14 \times 3} - \frac{45}{4 \times 15} = \frac{15-5}{15+5} + \frac{5}{4} - \frac{3}{4} =$

$\frac{2}{4} + \frac{5}{4} - \frac{3}{4} = 1$.

$\frac{26}{33}\frac{22}{81} = \frac{3 \times 23 \times 38}{3 \times 23 \times 49} = \frac{38}{49}$.

22. $\frac{240025}{10000} = \frac{5\times5\times9601}{5\times5\times400} = \frac{9601}{400}$.

$\frac{8125}{10000000} = \frac{5\times5\times5\times5\times13}{5\times5\times5\times5\times16000} = \frac{13}{16000}$.

$\frac{1.1214}{5.34} = \frac{1\,12.14}{534} = .21$.

$\frac{1121.4}{.534} = \frac{1121400}{534} = 2100$.

23. 7 cwt. 4 lb. = 788 lb.

3 t. 1 qr. = 6748 lb. ;

$\therefore$ fraction $= \frac{788}{6748} = \frac{197}{1687}$.

10 a. $= 10\times4840$ sq. yd. ;

$\therefore$ length of side $= \sqrt{48400}$ yd. $= 220$ yd.

Length of 4 sides $= 4 \times 220$ yd. $= 880$ yd.

$= \frac{1}{2}$ mile ;

$\therefore$ number of times $= 1 \div \frac{1}{2} = 2$.

24. The shares of all $= (15+3+10)$ seamen's shares ;

$\therefore$ 28 seamen's shares $=$ £399 7*s.* ;

$\therefore$ 1 " " $= \frac{\text{£399 7}s.}{28}$

$=$ £14 5*s.* 3*d.* ;

$\therefore$ 1 gunner's " $= 3\times($£14 5*s.* 3*d.*$)$

$=$ £42 15*s.* 9*d.* ;

$\therefore$ 1 lieutenant's " $= 10\times($£14 5*s.* 3*d.*$)$

$=$ £142 12*s.* 6*d.*

25.

```
1 | 67. | 96 | 16 (12.96
1
---
22 | 67
     44
---
249 | 2396
      2241
-----
2586 | 15516
       15516
```

$\sqrt{\frac{529}{2401}} = \frac{\sqrt{529}}{\sqrt{2401}} = \frac{23}{49}$

26. From midnight on Sunday to 6 p.m. on Wednesday is 66 hrs.

Time lost by the clock in 66 hrs. $= \frac{66 \times 4}{12}$ min. $= 22$min.;

$\therefore$ taking away the 10 min. already gained, the clock will indicate 12 min. to 6, or 5 h. 48 min.

27. Shortness in 22 yd. $= \frac{22 \times 5}{12}$ in. $= 9\frac{1}{6}$ in.,

$\therefore$ actual distance $= 22$ yd. $- 9\frac{1}{6}$ in.

$= 21$ yd. 2 ft. $2\frac{5}{6}$ in.

28. See Art. 174, 178, 181.

Interest $= \$(1900 \times 1\frac{3}{4} \times \frac{8}{100}) = \$266.$

Discount $= \$\frac{1900 \times 14}{114} = \$233.33\frac{1}{3}$;

$\therefore$ difference $= \$(266 - 233.33\frac{1}{3}) = \$32.66\frac{2}{3}$.

29. The interest $= \frac{2}{100}$ of \$170;

$\therefore$ the discount $= \frac{2}{102}$ of \$170;

$\therefore$ the P. W. $= \frac{100}{102}$ of \$170

$= \$166.66\frac{2}{3}$.

30. Interest $= \$(880 \times \frac{5}{4} \times \frac{9}{200}) = \$49.50.$

The interest $= \frac{45}{800}$ of \$929.50;

$\therefore$ the discount $= \frac{45}{845}$ of \$929.50

$= \$49.50.$

31. $\frac{\overset{7 \times 3 \times 3}{\cancel{2 \times 14}}}{\underset{6 \times 7}{\cancel{3 \times 2 \times 7}}} \times \frac{14}{9} = \frac{7 \times 3 \times 3 \times 6 \times 7 \times 14}{2 \times 14 \times 3 \times 2 \times 7 \times 9}$

$= \frac{7}{2} = 3\frac{1}{2}$.

32. $\frac{100005}{99000} \div \frac{55}{100} = \frac{100005}{990 \times 55} = \frac{6667}{3630}$.

33. Sum paid to produce \$1 income in the $3\frac{1}{2}$ per cents

$= \$\frac{91 \times 2}{7} = \$26.$

Sum paid to produce \$4 income in the $3\frac{1}{2}$ per cents

$= 4 \times \$26 = \$104$;

$\therefore$ the 4 per cents at 103 is the better investment.

Sum invested to produce a net income of 98 cents = \$26;

$\therefore$ " " " " \$4851

$$= \$\frac{4851 \times 26}{.98}$$

$$= \$128700.$$

34. Time required by faster vessel $= \frac{1200}{10}$ hr. $= 120$ hr.

" " slower " $= (120+36)$ hr.;

$\therefore$ the average rate of slower " $= \frac{1200}{156}$ mi.

$= 7\frac{9}{13}$ mi.

35. One gets 3 parts when the other gets 2 parts;

$\therefore \frac{3}{5}$ of \$87.50, or \$52.50 = one man's share,

and $\frac{2}{5}$ of \$87.50, or \$35 = the other "

36. Money realized by sale $= \$\frac{3430 \times 851}{100} = \$\frac{343 \times 171}{20}$.

First income $= \$\frac{3430 \times 7}{200} = \$120.05$.

Second " $= \$\frac{343 \times 171 \times 4}{20 \times 98} = \$119.70$;

$\therefore$ the difference = 35 cents.

37. Number of cubic feet in the cistern.

$$= \frac{93.75 \times 112 \times 16}{1000};$$

$\therefore$ the depth $= \frac{93.75 \times 112 \times 16}{1000 \times 8\frac{1}{2} \times 7}$ ft. = 3 ft.

38. Interest on \$330 for 2 mos. = \$3;

$\therefore$ " \$100 " 12 mos. $= \$\frac{100 \times 12 \times 3}{2 \times 330} = \$5\frac{5}{11}$.

Again, the interest on \$330 for 12 mos. $= 6 \times \$3 = \$18$;

$\therefore$ the discount off \$348 " = \$18;

$\therefore$ " \$333 for 12 mos.

$$= \$\frac{333 \times 18}{348} = \$17\frac{13}{58}.$$

39. (Feet in breadth)^2 $= 78^2 - 55^2$

$= (78 + 55)(78 - 55) = 133 \times 23$

$= 3059$;

$\therefore$ feet in breadth $= \sqrt{3059}$

$= 55.3\ldots$.

40. Area of floor $= (27\frac{1}{3} \times 20\frac{1}{6})$ sq. ft.
$= \frac{82 \times 121}{3 \times 6}$ sq. ft.;
$\therefore$ matting required $= \frac{9 \times 82 \times 121}{22 \times 3 \times 6}$ ft.
$= 225\frac{1}{2}$ ft. $= 75\frac{1}{6}$ yd.

41. $\frac{\frac{5 \times 9}{4 \times 5} - \frac{5 \times 6}{3 \times 5}}{\frac{7}{4} + \frac{17 \times 3}{34 \times 4}} \times \frac{17}{2} = \frac{\frac{9}{4} - \frac{6}{3}}{\frac{7}{4} + \frac{3}{8}} \times \frac{17}{2}$
$= \frac{2}{17} \times \frac{17}{2} = 1.$

42. Value of $\frac{2}{5}$ of $\frac{5}{4}$, or $\frac{1}{2}$ estate $= \$300$;
$\therefore$ " $\frac{5}{2} \times \frac{14}{5}$, or 7 estates $= \$(7 \times 2 \times 300)$
$= \$4200.$

43. $\frac{11}{13} + \frac{1}{15} = \frac{178}{195}$.
Part of cable on land $= 1 - \frac{178}{195} = \frac{17}{195}$;
$\therefore$ $\frac{17}{195}$ of cable $= 234\frac{2}{3}$ yd.;
$\therefore$ length of cable $= \frac{195 \times 704}{17 \times 3}$ yd. $= 2691\frac{13}{17}$ yd.

44.

	2		16 | 777 | 216
			8
6	5	1200	8777
		325	
		1525	7625
		25	
75	6	187500	1152216
		4536	
		192036	1152216

45. Selling price of $100s. = 110s.$
" " $15s. = \frac{15 \times 110}{100} s. = 16s.\ 6d.$

46. $800+756+404 = 1960$.

Number sent from Portsmouth $= \frac{800}{1960}$ of $490 = 200$.

" " Plymouth $= \frac{756}{1960}$ of $490 = 189$.

" " Sheerness $= \frac{404}{1960}$ of $490 = 101$.

47. *(a)* Compound interest $= 416\frac{2}{3} \times \$(1.08^2 - 1)$

$= \$69\frac{1}{3}$.

Simple " $= 416\frac{2}{3} \times \$0.16$

$= \$66\frac{2}{3}$;

$\therefore$ difference $= \$2\frac{2}{3}$.

(b) Interest on \$184 for $1\frac{1}{2}$ yr. $= \$27.60$;

$\therefore$ " \$100 for 1 yr. $= \$\frac{2 \times 100 \times 27.60}{3 \times 184}$

$= \$10$;

$\therefore$ the rate per cent. $= 10$.

48. \$180 = the amount of \$100;

$\therefore$ \$3213 = " $\$\frac{3213 \times 100}{180} = \$1785$.

49. Area of field = 30378240 sq. in.

Length of field = 9376 in.;

$\therefore$ breadth of field $= \frac{30378240}{9376}$ in. $= 3240$ in.

$= 270$ feet.

50. Area of walls $= \{(48 + 40) \times 14\frac{1}{4}\}$ sq. ft.

$= 1254$ sq. ft.

Deduction

$= \{(4 \times 8 \times 5\frac{1}{4}) + (2 \times 10 \times 6\frac{2}{3}) + (6\frac{1}{2} \times 5)\}$ sq. ft.

$= 333\frac{5}{6}$ sq. ft.;

$\therefore$ area of paper

$= (1254 - 333\frac{5}{6})$ sq. ft. $= 920\frac{1}{6}$ sq. ft.;

$\therefore$ length

$= \frac{920\frac{1}{6}}{2\frac{1}{2}}$ ft. $= \frac{5521}{45}$ yd.;

$\therefore$ cost $= \frac{5521}{45} \times 45$ far. $= 5521$ far.

$= £5\ 15s.\ 0\frac{1}{4}d.$

51. $\frac{\frac{8}{10} \text{ of } \frac{3}{5}}{\frac{10}{8} \text{ of } \frac{5}{3}} = \frac{8\times3\times8\times3}{10\times5\times10\times5} = \frac{144}{625}$.

52. $\frac{3}{500}$ of $25s + \frac{63}{900}$ of $100s. - \frac{8}{9}$ of $\frac{9}{4}s.$
$= 1d. + 7s. - 2s.$
$= 5s.\ 1d.$

53. $\frac{3}{8}$ of $\frac{7}{8}$ of $\frac{2}{7} = \frac{1}{6}$ = part whose value is required.
Value of $\frac{5}{7}$ of cargo = \$16000;
$\therefore$ " $\frac{1}{6}$ " $= \frac{1}{6}$ of $\frac{7}{5}$ of \$16000
$= \$3733\frac{1}{3}$.

54. Part mowed in 1 day by A, B, and C
$= (\frac{5}{3} + \frac{7}{9} + \frac{11}{12})$ a. $= \frac{121}{36}$ a.
Time to mow $\frac{121}{36}$ a. = 1 da.;
$\therefore$ " " 1 a. $= \frac{36}{121}$ da.;
$\therefore$ " " 121 a. $= \frac{121\times36}{121}$ da.
$= 36$ da.

55. Time it loses in $6\frac{1}{2}$ da.
$= 5$ m. 40 sec. $- 2$ m. 51 sec. $= 2$ m. 49 sec.
Time it loses in 1 da. $= \frac{2 \text{ m. } 49 \text{ sec.}}{6\frac{1}{2}} = 26$ sec.

56. Taxes $= \frac{14\frac{2}{5}}{100}$, or $\frac{72}{500}$ of rent;
$\therefore$ rent and taxes together $= (\frac{100}{100} + \frac{72}{500})$ of rent
$= \frac{572}{500}$ of \$720 $=$ \$823.68.

57. The 1st and 2nd pay $\frac{1}{3} + \frac{6}{5}$ of $\frac{1}{3}$, or $\frac{11}{15}$ of the bill;
$\therefore \frac{4}{15}$ of the bill $=$ \$2.50;
the bill $= \$\frac{15\times2.50}{4} = \$9.37\frac{1}{2}$.

58. Tax on \$1200 when it is half as much again = \$27;
$\therefore$ " \$750 " "
$= \$\frac{750\times27}{1200}$
$= \$16\frac{7}{8}$.

59. A's income $= \$\frac{552\times 3\frac{1}{2}}{92} = \$(6\times 3\frac{1}{2}) = \$19.50.$

B's " $= \$\frac{679\times 3}{97} = \$(7\times 3) = \$21$;

$\therefore$ difference $= \$21 - \$19.50 = \$1.50.$

60. Area of room in sq. yd. $= \frac{21\times 15\frac{1}{2}}{9}$;

$\therefore$ cost $= \frac{21\times 15\frac{1}{2}\times 42\frac{1}{2}}{9}$ cents $= \$15\frac{5}{9}.$

61. $\left(\frac{1\frac{1}{4}+1\frac{7}{5}}{2\frac{1}{5}+2\frac{1}{4}} + \frac{1\frac{1}{3}}{2\frac{1}{2}}\right) \times \left(\frac{2\frac{6}{11}}{1\frac{3}{5}} \div \frac{2\frac{9}{11}}{8\frac{7}{10}}\right) - \frac{281}{1405}$

$= \left(\frac{55+68}{84+105} + \frac{2\frac{2}{3}}{6\frac{2}{3}}\right) \times \left(\frac{20}{11}\times\frac{11\times 87}{29\times 10}\right) - \frac{1}{5}$

$= \frac{123+66}{189}\times 3 - \frac{1}{5} = 1\times 3 - \frac{1}{5} = 2\frac{2}{5}.$

62. Length of step $= \frac{3\frac{1}{2}\times 5280\times 12}{60\times 110}$ in. $= \frac{7\times 48\times 6}{60}$ in.

$= 33{\cdot}6$ in.

63. Length of street + length of column $= 8700$ ft.;

$\therefore$ time $= \frac{8700}{58\times 2\frac{1}{2}}$ min. $= 60$ min.

64. Area to be paved $= \{850\times(2\times 5\frac{1}{4})\}$ sq. ft.

$= (425\times 21)$ sq. ft.

Cost $= 425\times 21\times 37\frac{1}{2}$ cents

$= \$3346.87\frac{1}{2}.$

65. Part filled by one pipe in 1 hr. $= \frac{2}{3}$;

$\therefore$ " " the other " $= \frac{1}{3}$.

Time to fill $\frac{1}{3}$ of cistern $= 1$ hr.;

$\therefore$ " the cistern $= 3$ hr.

66. 27 men $=$ 54 boys.

Time for 54 boys $= 280$ hrs.;

$\therefore$ " 42 " $= \frac{54\times 280}{42}$ hr.

Number of hours in 1 day $= \frac{54\times 280}{45\times 42}$ hr.

$= 8$ hr.

67. Interest on \$125 for $1\frac{1}{2}$ yr. $= \$13.12\frac{1}{2}$;

$\therefore$ " \$100 " 1 yr. $= \$\frac{2\times 100\times 13.12\frac{1}{2}}{3\times 125}$

$= \$7.$

68. $5\frac{1}{2} \times \$6\frac{2}{3}$, or $\$\frac{110}{3}$ = interest on \$100;

$\therefore$ \$616 = " $\$\frac{616 \times 3 \times 100}{110}$

= \$1680.

69. Length cut off = $\{2\frac{1}{2} \div (1\frac{1}{3} \times 1\frac{1}{4})$ ft. = $1\frac{1}{2}$ ft.

Length remaining = $(18 - 1\frac{1}{2})$ ft. = $16\frac{1}{2}$ ft.

70. Area of room = $(20 \times 16\frac{3}{4})$ sq. ft. = 335 sq. ft.

Length bought for 168*s.* = $\frac{168}{34}$ yd. = 48 yd.

Area of carpet = $(48 \times \frac{3}{4})$ sq. yd. = 36 sq. yd.;

$\therefore$ part uncovered = $(335 - 9 \times 36)$sq. ft. = 11 sq. ft.

71. $\frac{\frac{3}{2} + \frac{7}{3}}{\frac{7}{3} + \frac{13}{4}} \div \frac{\frac{5}{7}}{1 + \frac{3}{7}} - \frac{6}{60}$

$= \frac{18+28}{28+39} \div \frac{1}{2} - \frac{6}{10}$

$= \frac{46}{67} \times \frac{2}{1} - \frac{1}{10}$

$= 1\frac{183}{670}$.

72. $\frac{2}{100}$ of 240*d.* + $\frac{3}{100}$ of 90*d.* + $\frac{13}{900}$ of 33*d.*

$= (4{\cdot}8 + 2{\cdot}7 + {\cdot}47\dot{6})d.$

$= 7{\cdot}97\dot{6}d.$

73. $\sqrt{30712{\cdot}5625} = 175{\cdot}25$; $\sqrt{\frac{625}{2401}} = \frac{25}{49}$;

and 175.25 of $\frac{25}{49} = 89\frac{81}{196}$.

$\sqrt{\frac{133225}{1000000000000}} = \frac{365}{1000000} = {\cdot}000365.$

74. They lose $\frac{100-37\frac{1}{2}}{100}$ of \$7850 = $\frac{125}{200}$ of \$7850

= \$4906.25.

75. Time for 14 men to mow 35 a. = 60 hr.;

$\therefore$ " 3 " " 24 a. = $\frac{24 \times 14 \times 60}{3 \times 35}$ hr.

Number of days = $\frac{24 \times 14 \times 60}{12 \times 3 \times 35}$

= 16.

76. 7 men and 9 women = 7 men and $\frac{81}{16}$ men, or $\frac{193}{16}$ men.

Time for 1 man to do the work $= 9 \times 144$ da.;

" $\frac{193}{16}$ men " " $= \frac{16 \times 9 \times 144}{193}$ da.

$= 107\frac{85}{193}$ da.

77. $\cdot 7 + \cdot 28 + \cdot 056 = 1 \cdot 036 = \frac{1036}{1000}$;

$\therefore$ A gets $\frac{\frac{7}{10}}{\frac{1036}{1000}}$, or $\frac{100}{148}$ of \$2849 $=$ \$1925.

B gets $\frac{\frac{28}{100}}{\frac{1036}{1000}}$, or $\frac{40}{148}$ of \$2849 $=$ \$770.

C gets $\frac{\frac{56}{1000}}{\frac{1036}{1000}}$, or $\frac{8}{148}$ of \$2849 $=$ \$154.

78. \$(400 − 360) $=$ the interest on \$360; (Art. 181)

$\therefore$ \$400 $=$ " $\$\frac{400 \times 360}{40}$

$=$ \$3600.

Again, the interest on \$360 for 2 yr. $=$ \$40;

$\therefore$ " \$100 for 1 yr. $= \$\frac{100 \times 40}{2 \times 360}$

$= \$5\frac{5}{9}$.

79. Selling price of 100 oranges $= \frac{100 \times 12}{8}$ cents

$=$ \$1.50.

Loss on an outlay of \$2.50 $=$ \$1;

$\therefore$ " " \$100 $= \$\frac{100 \times 1}{2.50}$

$=$ \$40.

80. Area of walls $= (72 \times 11)$ sq. ft. $= 792$ sq. ft.

Area of windows $= (2 \times 9 \times 3)$ sq. ft. $= 54$ sq. ft.

Area of door $= (7 \times 3\frac{1}{2})$ sq. ft. $= 24\frac{1}{2}$ sq. ft.

Area of fireplace $= (4 \times 4\frac{1}{2})$ sq. ft. $= 18$ sq. ft.

Area to be papered $= (792 - 54 - 24\frac{1}{2} - 18)$ sq. ft.

$= 695\frac{1}{2}$ sq. ft.;

$\therefore$ length of paper $= (695\frac{1}{2} \div 2\frac{1}{4})$ ft. $= \frac{2782}{9}$ ft.;

$\therefore$ cost $= (\frac{2782}{9} \times \frac{9\frac{1}{2}}{36})s.$

$=$ £4 1s $6\frac{23}{71}d.$

81. (1) $\frac{10+\frac{4}{5}+\frac{1}{11}-\frac{1}{2}}{15+\frac{1}{5}+\frac{7}{11}-\frac{1}{4}} = \frac{10\frac{43}{110}}{15\frac{129}{220}} = \frac{1143\times 2}{3429} = \frac{2}{3}$.

(2) $\frac{1.802\times 7.03}{\frac{20}{9}-\frac{74}{333}} = \frac{12.666806}{2} = 6.33403$.

82. Part sold $= \frac{135}{999}$, or $\frac{5}{37}$ of his share,

$\therefore$ part remaining $= \frac{32}{37}$ of his share

$= \frac{32}{37}$ of $\frac{3}{16}$

$= \frac{6}{37}$.

83. Part done by A, 2 B's, and C in 1 da. $= \frac{1}{8}+\frac{1}{12}$.

" " A, B, and C " $= \frac{1}{6}$;

$\therefore$ " " 2 A's, 2 B's, 2C's " $= \frac{1}{3}$;

$\therefore$ " " A and C " $= \frac{1}{3}-(\frac{1}{8}+\frac{1}{12})$

$= \frac{1}{8}$;

$\therefore$ time required by A and $C = 8$ days.

84. Number of hours between midnight on Sunday to 4 p.m. Wednesday $= 64$.

Time gained in 24 hr. $= 7\frac{1}{2}$ min.;

$\therefore$ " " 64 hr. $= \frac{64\times 7\frac{1}{2}}{24}$ min.

$= 20$ min.

Hence the time on Wednesday is 4 hr. 32 min.

85. $33+7+5 = 45$.

Number of lb. of nitre $= \frac{33}{45}$ of 30 lb.

$= 22$ lb.

" " charcoal $= \frac{7}{45}$ of 30 lb.

$= 4\frac{2}{3}$ lb.

" " sulphur $= \frac{5}{45}$ of 30 lb.

$= 3\frac{1}{3}$ lb.

86. Interest $= \$\left(1639 \times \frac{4\frac{3}{4}}{12} \times \frac{6\frac{3}{19}}{100}\right) = \$39.95\frac{1}{16}$.

Discount $= \$(1639 \times \frac{119}{4917}) = \$39$;

$\therefore$ difference $= \$.95\frac{1}{16}$.

87. The bank discount $= \$(10400 \times \frac{6}{12} \times \frac{8}{100}) = \$416.$
The true " $= \$(10400 \times \frac{4}{104}) = \$400;$
$\therefore$ difference $= \$16.$

88. Part sold at cost $= \frac{1}{8}$ of $\frac{1}{5}$, or $\frac{1}{40}$ of goods.
" $\frac{1}{3}$ of cost $= \frac{7}{8}$ of $\frac{1}{5}$, or $\frac{7}{40}$ "
Price of goods realized $= (\frac{1}{40} + \frac{1}{3}$ of $\frac{7}{40})$ of cost
$= \frac{1}{12}$ of cost;
$\therefore$ cost of goods $= 12 \times \$1155,$
and loss $= 11 \times \$1155$
$= \$12705.$

89. The gallon contains $\frac{277 \cdot 274}{1728}$ cub. ft.;
$\therefore$ the gallon weighs $\frac{277 \cdot 274 \times 1000}{1728}$ oz.;
$\therefore$ the pint weighs $\frac{277274}{16 \times 8 \times 1728}$ lb. $= 1 \cdot 2535 \ldots$ lb.

90. Area of floor $= (22\frac{1}{2} \times 20\frac{1}{4})$ sq. ft. $= \frac{45 \times 81}{8}$ sq. ft.;
$\therefore$ cost of carpet $= \frac{45 \times 81}{9 \times 8} \times \$1.20 = \$60.75.$
Area of walls $= \{(45 + 40\frac{1}{2}) \times 10\frac{3}{4}\}$ sq. ft.
$= \frac{171 \times 43}{8}$ sq. ft.;
$\therefore$ cost of paper $= \frac{171 \times 43}{9 \times 8} \times 20$ cents
$= \$20.42\frac{1}{2}.$

91. $\dfrac{40 \div 5}{2 \cdot 4232323 + 3 \cdot 5765765 + 2 \cdot 0001911} = \frac{8}{8} = 1.$

92. See Ex. paper X., example 5.

93. £·60625 = 12*s.* $1\frac{1}{2}$*d.*

·142857 of 14*s.* $10\frac{1}{2}$*d.* $= \frac{1}{7}$ of 14*s.* $10\frac{1}{2}$*d.* $= 2s.\ 1\frac{1}{2}d.$
$\frac{23}{11}$ of $\frac{3}{71}$ of £3 5*s.* 1*d.* $= \frac{23}{11}$ of $\frac{3}{71}$ of 781*d.* $= 5s.\ 9d.$
Sum $= 20s.$
Also $20s. = \frac{20}{27}$ of $27s.$
$\frac{20}{27} = \cdot 740.$

94. Time gained in $7\frac{1}{2}$ hr. $= \left\{7\frac{1}{2} \times (3\frac{1}{2} \div 24)\right\}$ min.
$= 1\frac{3}{32}$ min.;
$\therefore$ it must be set at $1\frac{3}{32}$ min. to 12.

95. Interest $= \$(956.25 - 750) = \$206.25$;
$\therefore$ interest on \$750 for $3\frac{2}{3}$ yr. $= \$206.25$;
$\therefore$ " \$100 " 1 yr. $= \$\frac{5 \times 100 \times 206.25}{11 \times 750}$
$= \$7\frac{1}{2}$

96. 1 per cent on \$5420 gives \$54.20;
$\therefore$ income at the lower rate on \$9970 $= \$(453 - 54.20)$
$= \$398.80$;
$\therefore$ " " " \$100
$= \$\frac{100 \times 398.80}{9970}$, or \$4;
hence the rates are 4 % and 5 %.

97. Area of wall $= \left\{540 + 184\frac{1}{2}) \times 8\frac{1}{3}\right\}$ sq. ft.
$= \frac{1449 \times 25}{9 \times 6}$ sq. yd.;
$\therefore$ cost of wall $= \frac{1449 \times 25}{9 \times 6} \times \$1.20$
$= \$805.$

98. Income from £75 invested $=$ £3.
Money got from £75 " $=$ £78.
Income from £78 " $= £\frac{78 \times 8}{208}$
$=$ £3.

99. L. C. M. of 2, 3, 4, 5, 6 $= 60$.
We must now find the least multiple of 60 which is a perfect square. This is 900.

100. The interest on \$320 for 8 mo. $= \$40$;
$\therefore$ " " \$360 for 12 mo.
$= \$\frac{360 \times 12 \times 40}{320 \times 8}$
$= \$67.50.$

101.
$$\begin{array}{r} 57875 \\ (729)(81)(9) \\ \hline 520875 = 9\times 57875. \\ 4687875 = 9\times 520875. \\ 42190875 = 9\times 4687875. \\ \hline 42238274625. \end{array}$$

$$\begin{array}{r|l} 9 & 123456 \\ \hline 7 & 13717 \text{ and } 3 \text{ units over.} \\ \hline & 1959 \text{ and } 4 \text{ groups of } 9 \text{ units each over;} \end{array}$$

$\therefore$ quotient $= 1959\frac{39}{63}$.

102. 1 metre $= 1{\cdot}0936$ yd.;

$\therefore$ 1 centimetre $= {\cdot}010936$ yd.

$= ({\cdot}010936\times 36)$ in.

$= {\cdot}393696$ in.

103. Part done by 2 A, B and C daily $= \frac{1}{4} + \frac{5}{23}$.

" " B and C " $= \frac{4}{23}$;

$\therefore$ " " 2 A " $= \frac{1}{4} + \frac{1}{23}$

$= \frac{27}{92}$;

$\therefore$ A can do the work in $\frac{184}{27}$ da. $= 6\frac{22}{27}$ da.

Part done by B daily $= \frac{1}{4} - \frac{27}{184}$

$= \frac{19}{184}$;

$\therefore$ B can do the work in $\frac{184}{19}$ da. $= 9\frac{13}{19}$ da.

Part done by C daily $= \frac{4}{23} - \frac{19}{184}$

$= \frac{13}{184}$;

$\therefore$ C can do the work in $\frac{184}{13}$ da. $= 14\frac{2}{13}$.

104. M has 12 miles start.

N gains 4 miles per hour, and hence would overtake M in 3 hours.

When N arrives M has 4×6 miles to go.

It requires $N\frac{4\times 6}{4}$ hr., or 6 hr. to gain this distance on M.

Hence M travels $(5+6+4)$ hr. and goes 15×6 miles, or 90 miles.

105: Interest $= \$(2733\frac{1}{3}\times 3\frac{3}{4}\times \frac{4}{100})$
$= \$410.$

Amount of \$1 at compound interest $= \$1.157625$;

$\therefore$ sum required $= \$\frac{926.10}{1.157625}$
$= \$800.$

106. Discount off $\$108\frac{1}{6} = \$8\frac{1}{6}$;

$\therefore$ " " $\$1622.50 = \$\frac{1622.50\times 8\frac{1}{6}}{108\frac{1}{6}}$
$= \$122.50.$

Interest on $\$1760 = \$(1760\times\frac{5}{4}\times\frac{6}{100})$
$= \$132;$
$\therefore$ difference $= \$9.50.$

107. Cost of 1 apple of 1st kind $= \frac{1}{3}d.$
" " 2nd " $= \frac{1}{2}d.;$
$\therefore$ average cost of 1 apple $= \frac{\frac{1}{3}+\frac{1}{2}}{2}d.$
$= \frac{5}{12}d.$
Selling price of 1 apple $= \frac{2}{5}d.;$
$\therefore$ loss on an outlay of $\frac{5}{12}d. = (\frac{5}{12}-\frac{2}{5})d.$
$= \frac{1}{60}d.;$
$\therefore$ " " $100d. = \frac{100\times\frac{1}{60}}{\frac{5}{12}}d.$
$= 4d.$

108. What he sold for \$91 he should sell for \$107;
$\therefore$ " " \$182 " "
$= \$\frac{182\times 107}{91}$
$= \$214.$

109. Area of walls $= \left\{(28\frac{5}{6}+27\frac{1}{6})\times 12\frac{1}{4}\right\}$ sq. ft.
$= 686$ sq. ft.
Deduction $= (48+20+13\times 2\frac{5}{12})$ sq. ft
$= 99\frac{5}{12}$ sq. ft.
Area of paper $= 586\frac{7}{12}$ sq. ft.
Cost " $= \frac{7039\times 72}{9\times 12\times 5}$ cents
$= \$9.38\frac{8}{15}$.

110. Contents of two longer sides
$= (2\times 4\times 2\times \frac{1}{12})$ c. ft. $= \frac{4}{3}$ c. ft.
Contents of two shorter sides
$= (2\times 2\frac{5}{6}\times 2\times \frac{1}{12})$ c. ft. $= \frac{17}{18}$ c. ft.
Contents of bottom
$= (3\frac{5}{6}\times 2\frac{5}{6}\times \frac{1}{12})$ c. ft. $= \frac{391}{432}$ c. ft.;
$\therefore$ whole contents $= \frac{576+408+391}{432}$ c. ft. $= \frac{1375}{432}$ c. ft.;
$\therefore$ cost $= \frac{12}{11}$ of $\frac{1375\times 9}{432\times 27}s. = \frac{125}{108}s. = 1s.\ 1\frac{8}{9}d.$

111. We are required to find the L. C. M. of 1, 2, 3, 4, 5, 6, 7, 8;

L. C. M. $= 840$.

Hence the bells will be tolling together in 840 sec., or 14 min.

112. $\frac{1\frac{3}{4}-\frac{3}{4}}{\frac{1}{2}+5\frac{1}{2}} \div \frac{1}{6} - \left\{\frac{10}{21}+\frac{4}{21}-\frac{10}{63}\right\} \div \frac{4}{7}$
$= \frac{1}{6} \div \frac{1}{6} - \frac{32}{63}\times\frac{7}{4} = 1 - \frac{8}{9} = \frac{1}{9}$.

113. 2 per cent. of A's capital $= \$220$;
$\therefore$ A's capital $= \frac{100\times 220}{2} = \$11000$;
$\therefore$ B and C's capital $= \frac{3}{2}$ of $\$11000$
$= \$16500$;
$\therefore$ the capital of each $= \frac{\$16500}{2} = \$8250$.

114. Since the fast train goes as far in 5 hr. as the slow one does in 6 hr., the rates are as 5 : 6.

Since the fast train gains 10 miles in 2 hours, it gains 5 mi. in 1 hr.

Hence $\frac{6}{5}$ of rate of slow train = rate of slow train + 5 mi.;

$\therefore$ $\frac{1}{5}$ " " " = 5 mi.

and " " " = 5 × 5 mi.

= 25 mi.

Rate of fast train = (25 + 5) mi.

= 30 mi.

115. Income on £80 annually = £6;

$\therefore$ " £100 " = $£\frac{100 \times 6}{80} = £7\frac{1}{2}$.

Amount of Turkish stock = $£\frac{5000 \times 100}{80}$;

money from sale of stock

$= £\frac{5000 \times 100 \times 104}{80 \times 100}$

= £6500.

Income from £90 invested in railway shares

= £4½;

$\therefore$ new income = $£\frac{6500 \times 4\frac{1}{2}}{90}$ = £325.

First income = $£\frac{5000 \times 6}{80}$ = £375;

$\therefore$ he has £50 less income.

116. 30 men and 10 boys reap 130 a. in 4 da.

14 men and 10 boys " 66 a. " 4 da.;

$\therefore$ 16 men reap 64 a. in 4 da.;

$\therefore$ 1 man reaps 1 a. in 1 da.

But 6 men and 2 boys reap 13 a. in 2 da.;

$\therefore$ 2 boys reap 1 a. in 2 da.;

$\therefore$ 1 boy reaps ½ a. in 1 da.,

and 2 men and 2 boys reap 2½ a. in 1 da.;

" " " 10 a. in $\frac{10 \times 1}{2\frac{1}{2}}$

= 4 days.

117. Retail price $= (\frac{100}{100} + \frac{30}{100})$ of cost price
$= \frac{130}{100}$ of \$4.75
$= \$6.17\frac{1}{2}$.

118. First interest $= \$(625 \times \frac{8}{12} \times \frac{7}{100})$
$= \$29.16\frac{2}{3}$.

Second " $= \$(1093.75 \times \frac{4}{12} \times \frac{8}{100})$
$= \$29.16\frac{2}{3}$.

119. Time when the difference is 6 min. = 12 hr. ;
∴ " " " $16\frac{1}{2}$ "
$= \frac{16\frac{1}{2} \times 12}{6}$ hr.
$= 33$ hr.

33 hr. from noon on Monday is 9 p.m. Tuesday.
Time gained by the fast goer in 33 hr.
$= \frac{33 \times 4}{12}$ min.
$= 11$ min. ;

hence it will indicate 9 hr. 11 min.
Time lost by the slow goer in 33 hr.
$= \frac{33 \times 4}{24}$ min.
$= 5\frac{1}{2}$ min. ;

hence it will indicate $5\frac{1}{2}$ min. to 9, or 8 hr. 54 min. 30 sec.

120. Area of each grass plot $= (66 \times 36)$ sq. ft. ;
∴ " covered by grass $= \frac{4 \times 66 \times 36}{9}$ sq. yd.
$= 1056$ sq. yd.
Area of whole court $= (50 \times 30)$ sq. yd.
$= 1500$ sq. yd.
Cost of grass $= \$(1056 \times .70)$
$= \$739.20$.
Cost of stones $= \$(444 \times 9 \times .12\frac{1}{2})$
$= \$499.50$;
∴ total cost $= \$(739.20 + 499.50)$
$= \$1238.70$.

121. Number of leap years in 400 consecutive years
$= 97$. Art. 151.
Number of times the 29th occurs in an ordinary year
$= 11$;
hence in 400 years it will occur $400 \times 11 + 97$
$= 4497$

122. Since $62\frac{1}{2}$ cents $= \frac{5}{8}$ of dollar, he received $\frac{5}{8}$ of the debt;

$\therefore$ $\frac{5}{8}$ of debt $= \$281.25$,
and " $= \$\frac{8 \times 281.25}{5}$
$= \$450$.

123. The shares are in the ratio of $1 \times 4 \times 10$.
$2 \times 3 \times 12$, and $3 \times 1 \times 20$.
$1 \times 4 \times 10 = 40$.
$2 \times 3 \times 12 = 72$.
$3 \times 1 \times 20 = 60$.
172.
A's share $= \frac{40}{172}$ of $\$43 = \$10$.
B's " $= \frac{72}{172}$ of $\$43 = \$18$.
C's " $= \frac{60}{172}$ of $\$43 = \$15$.

124. The cubic content of a brick in the second case
$= (\frac{4}{5})^3$ of that of a brick in the first case.
Hence we may leave the exact dimensions of the first brick out of account and find the cost thus:
cost of 1 brick $= \$(\frac{64}{125}$ of $\frac{213.50}{9760})$;
$\therefore$ " 100 bricks $= \$(100 \times \frac{64}{125} \times \frac{213.50}{9760})$
$= \$1.12$.

125. Number of years $= 100 \div 3\frac{1}{3}$
$= 30$.

126. At 11 o'clock the hands are 5 minute-spaces

apart, and as the minute hand moves over 12 minute-spaces while the hour hand moves over 1, they will be an exact number of minute-spaces apart at 12 min. past 11. For the same reason they will be an exact number of minute-spaces apart at 11 hr. 24 min., at 11 hr. 36 min., and at 11 hr. 48 min. Therefore, they will be an exact number of minute-spaces apart 4 times between 11 and 12.

127. Time A walks 20 mi. $= 20 \times 11$ min.
$= 220$ min.
" B " " $= (220 + 45)$ min.
$= 265$ min.
$\therefore$ B's rate $= \frac{265}{20}$ min. per mile
$= 13\frac{1}{4}$ min. "
Again, time A walks 5 mi. $= 55$ min.
" B " " $= 66\frac{1}{4}$ min.;
$\therefore$ A wins by $11\frac{1}{4}$ min.
$\therefore$ Distance he walks in $11\frac{1}{4}$ min. $= \frac{11\frac{1}{4}}{13\frac{1}{4}}$ mi.
$= \frac{45}{53}$ mi.

128. $\frac{1}{4} \times 3 + \frac{1}{2} \times$ (number of months when the remainder should be paid) $= 4\frac{1}{2}$;
$\therefore$ $\frac{1}{2} \times$ (number of months) $= 4\frac{1}{2} - \frac{3}{4}$,
and number of months $= 2 \times 3\frac{3}{4}$
$= 7\frac{1}{2}$.

129. Buying price $= \frac{100}{110}$ of 99.
$= 90$.
Income $= \$\frac{15345 \times 3}{99}$
$= \$465$.

130. Cubic contents of tank $= (8 \times 5\frac{1}{3} \times 4\frac{1}{2})$ c. ft.
$= 192$ c. ft.
Weight of water $= \frac{192 \times 1000}{16}$ lb.
$= 12000$ lb.

$$\text{Number of gallons} = \frac{12000 \times 4}{5 \times 8}$$
$$= 1200.$$

131. $$\frac{1}{\frac{29 \times 36}{4 \times 11} + \frac{36}{11}} \div \left(\frac{27-26}{117}\right) - \frac{27}{6} \div \frac{2 \times 3 \times 63}{3 \times 8}$$
$$= \frac{11}{297} \times \frac{117}{1} - \frac{27 \times 4}{6 \times 63} = \frac{117}{27} - \frac{2}{7} = 4\frac{1}{21}.$$

132. $\frac{11}{23} + \frac{2}{3}$ of $\frac{12}{23} = \frac{11}{23} + \frac{8}{23} = \frac{19}{23}$;
hence $\frac{4}{23}$ of the boys $= 8$,
and number of boys $= \frac{23 \times 8}{4}$
$= 46.$

133. The L. C. M. of 8 and 10 $= 40$.
In 40 ft. one wheel makes 5 revolutions and the other 4;
$\therefore$ distance required $= 100 \times 40$ ft.
$= 4000$ ft.

134. A works $(4\frac{1}{2} + 3\frac{1}{2})$ hr., or 8 hr.; B works $4\frac{1}{2}$ hr.
Cost of $12\frac{1}{2}$ hr. work $= \$1.37\frac{1}{2}$;
$\therefore$ " 8 " $= \$\frac{8 \times 1.37\frac{1}{2}}{12\frac{1}{2}}$
$= \$0.88.$
" $4\frac{1}{2}$ " $= \$\frac{4\frac{1}{2} \times 1.37\frac{1}{2}}{12\frac{1}{2}}$
$= \$0.49\frac{1}{2}.$

135. When the minute hand is between 2 and 3;
number of min. past $3 + \frac{1}{12}$ (number of min. past 3) $= 15$;
$\therefore$ $\frac{13}{12}$ of number of min. past 3 $= 15$;
$\therefore$ number of min. past $3 = \frac{12 \times 15}{13} = 13\frac{11}{13}$.

Again, when the hands are together between the figures 3 and 4, the number of min. past $3 - \frac{1}{12}$ (number of min. past 3) $= 15$;
$\therefore$ $\frac{11}{12}$ of number of min. past 3 $= 15$;
$\therefore$ number of min. past $3 = \frac{12 \times 15}{11} = 16\frac{4}{11}$.

136. Time for \$320 to gain \$24 interest $= 1$ yr. ;

$\therefore$ " " " \$320 " $= \frac{320 \times 1}{24}$ yr.

$= 13\frac{1}{3}$ yr.

137. Present value $= £\frac{2358 \times 100}{108}$.

Income from £144 invested $= £9$;

$\therefore$ " " $£\frac{2358 \times 100}{108}$ " $= £\frac{2358 \times 100 \times 9}{108 \times 144}$

$= £136\ 9s.\ 2d.$

Also rate per cent. $= \frac{100 \times 9}{144}$

$= 6\frac{1}{4}$.

138. Since A can make 50 when B makes 45,

and A " " 50 " C " 40 :

$\therefore$ B " " 90 " C " 80 :

$\therefore$ B can give C 10 points.

139. Sum invested for £3, income $= £90$;

$\therefore$ " " " £2000, " $= £\frac{2000 \times 90}{3}$

$= £60000.$

140. 14 mi. 6 fur. $= (236 \times 330)$ ft.

Amount of water drawn from Canal

$= (236 \times 330 \times 48 \times 1\frac{1}{2})$ c. ft.

Amount of water in the lock

$= (80 \times 12 \times 8\frac{1}{2})$ c. ft. ;

$\therefore$ number of barges $= \frac{236 \times 330 \times 4}{80 \times 0 \times 17}$

$= 38.$

141. $\frac{\frac{22}{7}}{\frac{39}{5} \times \frac{3}{4}} \times \frac{\frac{13}{28}}{\frac{45}{56}}$ of \$5.67

$= \$\frac{22 \times 5 \times 4 \times 13 \times 56 \times 5.67}{7 \times 39 \times 3 \times 28 \times 45}$

$= \$1.76.$

142. Work done by A in 10 da. $= \frac{10}{6}$.

" destroyed by B " $= \frac{5}{4}$;

$\therefore$ part of work done $= \frac{10}{6} - \frac{5}{4}$

$= \frac{5}{12}$.

" " to be done $= \frac{7}{12}$.

Time required by A to finish $= (\frac{7}{12} \div \frac{1}{6})$ da.

$= 3\frac{1}{2}$ da.

143. Let 1 represent the quantity of water in each cistern; then,

quantity of water which runs out of first cistern in 1 hr. $= \frac{1}{5}$;

$\therefore$ " " " in no. of hr. required $= \frac{\text{no. of hours}}{5}$.

" " " second cistern " $= \frac{\text{no. of hours}}{4}$.

By conditions of question,

$$1 - \frac{\text{no. of hours}}{5} = 2\left(1 - \frac{\text{no. of hours}}{4}\right)$$

$$= 2 - \frac{\text{no. of hours}}{2};$$

$$\therefore \frac{3 \text{ no. of hours}}{10} = 1,$$

$$\text{and no. of hours} = \frac{10}{3}$$

$$= 3\frac{1}{3}.$$

144. In 1 day a man does $\frac{1}{180}$ of work; a woman, $\frac{1}{240}$ of it; a boy, $\frac{1}{300}$ of it; a girl, $\frac{1}{360}$ of it;

$\therefore$ 1 man, 2 women, 3 boys, and 4 girls do $(\frac{1}{180} + \frac{2}{240} + \frac{3}{300} + \frac{4}{360})$ of work daily.

Time to do all the work

$= \{1 \div (\frac{1}{180} + \frac{2}{240} + \frac{3}{300} + \frac{4}{360})\}$ da.

$= \frac{3600}{126}$ da.

$= 28\frac{4}{7}$ da.

145. The fast train runs 5 miles while the slow one runs 3 miles ;

$\therefore$ distance run by the slow train $= \frac{3}{5}$ of distance run by quick train.

But distance run by quick train = distance run by slow one + 100 miles ;

$\therefore$ distance run by quick train $= \frac{3}{5}$ of distance run by quick one + 100 miles ;

$\therefore$ $\frac{2}{5}$ of distance run by quick train $= 100$ miles,

and " " " $= \frac{5 \times 100}{2}$ mi.

" " " $= 250$ mi.

" " slow " $= \frac{3}{5}$ of 250 mi.

$= 150$ mi. ;

$\therefore$ distance between London and Edinburgh

$= (250 + 150)$ mi. $= 400$ mi.

146. Price of 3 per cents. $= 75$.

$\therefore$ " $3\frac{1}{2}$ " $= \frac{3\frac{1}{2} \times 75}{3}$

$= 87.5$.

147. $(2\cdot\dot{3} + 1\cdot1\dot{5} + \cdot5\dot{2}\dot{4}) = \frac{3973}{990}$;

$\therefore$ A gets $\frac{2\cdot\dot{3}}{\frac{3973}{990}}$ of \$1986.50 $=$ \$1155.

B gets $\frac{1\cdot1\dot{5}}{\frac{3973}{990}}$ of \$1986.50 $=$ \$572.

C gets $\frac{\cdot5\dot{2}\dot{4}}{\frac{3973}{990}}$ of \$1986.90 $=$ \$259.50.

148. £1 $=$ 11 guilders 12 kreut. $= 672$ kreut.

£1 $= 25.5$ fr. $= \frac{25.5}{20} \times 560$ kreut.

$= 714$ kreut.

Gain on 672 kreut. $= 42$ kreut ;

$\therefore$ " 100 kreut. $= \frac{100 \times 42}{672}$ kreut.

$= 6\frac{1}{4}$ kreut.

149. 35 yards = 32 metres;

$\therefore$ $69\frac{1}{2}$ miles $= \frac{69\frac{1}{2}\times 8\times 40\times 5\frac{1}{2}\times 32}{35}$ metres.

$= 111835\frac{3}{7}$ metres.

150. Area of walls $= (2\times 36\times 14)$ sq. ft. $= 1008$ sq. ft.

Deduction $= (2\times 8\times 4+3\times 10\times 5)$ sq. ft.

$= 214$ sq. ft.

Area to be painted $= 794$ sq. ft.

Cost of 50 sq. ft. $=$ £2 16*s.* 3*d.*;

$\therefore$ " 794 sq. ft. $= \frac{794\times(\text{£}2\ 16s.\ 3d.)}{50}$,

$=$ £44 13*s.* 3*d.*

Area painted for $56\frac{1}{4}s.$ $= 50$ sq. ft.;

$\therefore$ " " $9s.$ $= \frac{9\times 50}{56\frac{1}{4}}$ sq. ft.

$= 8$ sq. ft.;

$\therefore$ additional height $= \frac{8}{4\times 18}$ ft.;

$= \frac{1}{9}$ ft.

151. $\frac{\frac{9}{4}}{\frac{8}{3}} + \frac{7\frac{7}{10}}{12\frac{5}{6}} + \frac{1}{2} + \frac{9}{160}$

$= 3\frac{7}{32} + \frac{77\times 6}{77\times 10} + \frac{1}{2} + \frac{9}{160}$

$= 3\frac{7}{32} + \frac{3}{5} + \frac{89}{160}$

$= 2.$

152. $41{\cdot}06328 \div {\cdot}0438 = 937$, and ${\cdot}02268$ over;

$\therefore$ there are 937 lines, and the length of the remainder is ${\cdot}02268$ in.

153. Distance A runs in 1 min. $= (2\frac{1}{3}\div 16\frac{4}{5})$ mi.

$= (\frac{7}{3}\times\frac{5}{84})$ mi.;

$\therefore$ " " 34 " $= (34\times\frac{7}{3}\times\frac{5}{84})$ mi.,

and " B " 34 "

$= (1\frac{8}{17}\times 34\times\frac{7}{3}\times\frac{5}{84})$mi.

$= 5$ mi.;

$\therefore$ length of course $= (2\frac{1}{3}+5)$ mi.

$= 7\frac{1}{3}$ mi.

154. Rate of boat—rate of stream $= 6 \times 1\frac{1}{4}$ mi. per hr.
$= 7\frac{1}{2}$ mi. "
Rate of stream $= 2$ mi. ";
$\therefore$ rate of boat in still water $= 9\frac{1}{2}$ mi. ";
$\therefore$ rate of boat in usual state of stream
$= 9$ mi. ".
Time to go 9 mi. $= 60$ min.;
$\therefore$ " " $1\frac{1}{4}$ mi. $= \frac{1\frac{1}{4} \times 60}{9}$ min.
$= 8\frac{1}{3}$ min.

155. 5 % is $12d.$ in the £;
$\therefore$ he has $240d. - (10+12)d.$ left out of £1.
$\frac{218}{240}$ of original income $=$ £545
$\therefore$ " " $= £\frac{240 \times 545}{218}$
$= £600.$

156. Net income from $\$(107\frac{1}{2} + \frac{1}{8}) = \$(6 - \frac{1}{20}$ of $6)$;
$\therefore$ " " \$14350 $= \$\frac{14350 \times 5.70}{107\frac{1}{8}}$
$= \$760.$

157. He may ride for $\frac{1}{3}$ of 5 hours, because he can then walk back in $\frac{2}{3}$ of 5 hours.;
$\therefore$ he may ride $\frac{2}{3} \times 10$ mi. $= 16\frac{2}{3}$ mi.

158. Call the place where the trains meet M;
the distance from L to $M = 4 \times$ rate of slow train in miles per hour;
$\therefore$ distance from N to $M = 1 \times$ rate of quick train in miles per hour;

$$\therefore \frac{4 \times \text{rate of slow train}}{1 \times \text{rate of quick train}} = \frac{\text{distance from } L \text{ to } M}{\text{distance from } N \text{ to } M};$$

$$\text{but } \frac{\text{rate of slow train}}{\text{rate of quick train}} = \frac{\text{distance from } N \text{ to } M}{\text{distance from } L \text{ to } M};$$

$\therefore$ compounding the ratios (Art. 215),

$$\frac{4 \times (\text{rate of slow train})^2}{(\text{rate of quick train})^2} = 1;$$

$\therefore$ 2 × (rate of slow train) = rate of quick train.

159. Since £170 = 4233 fr.;

$\therefore$ £1 $= \frac{4233}{170}$ fr.

$= 24{\cdot}9$ fr.

Again, £400 = 503 × 20 fr.;

$\therefore$ £1 $= \frac{503 \times 20}{400}$ fr.

$= 25{\cdot}15$ fr.

160. The cube root of $50{\cdot}653 = 3{\cdot}7$;

$\therefore$ length of outside edge $= (12 \times 3{\cdot}7 + 2 \times 1{\cdot}\dot{3})$in.

$= 47\frac{1}{15}$ in.

161. $\frac{45}{7} \times \frac{62-55}{152-89} \times \frac{2}{1\frac{3}{7}}$

$= \frac{45}{7} \times \frac{7}{63} \times \frac{14}{10}$

$= 1.$

162. Since £$3\frac{9}{10}$ = 1 oz. Troy;

$\therefore$ £423267 $= \frac{423267 \times 1}{3{\cdot}9}$ oz. Troy

$= \frac{423267 \times 1 \times 20 \times 24}{7000 \times 3{\cdot}9}$ lbs. avoir.

$= 7442\frac{27}{35}$ lb.

163. Part done by A in 1 hr. $= \frac{1}{2}$.

" " B " $= \frac{3}{8}$.

" " C " $= \frac{3}{8}$;

$\therefore$ " " A, B and C in 1 hr.

$= \frac{1}{2} + \frac{3}{8} + \frac{3}{8}$

$= \frac{10}{8}$;

$\therefore$ time to do the work $= \frac{8}{10}$ hr.

$= 48$ min.

164. Interest $= \$771.09\frac{3}{8} - \$750 = \$21.09\frac{3}{8}$.
Time for which \$56.25 is interest on \$750 $=$ 12 mo.;
$\therefore$ time for which $\$21.09\frac{3}{8}$ is interest on \$750
$$= \frac{21.09\frac{3}{8} \times 12}{56.25} \text{ mo.}$$
$$= 4\frac{1}{2} \text{ mo.}$$

165. 5 parts in 20 parts in the 1st glass are spirit.
4 " " " 2nd " " ;
$\therefore$ 9 " 40 of the mixture are spirit;
$\therefore$ the ratio is 9 of spirit to 31 of water.

166. Selling price of \$100 $=$ \$125;
$\therefore$ $\frac{80}{100}$ of marked price $=$ \$125;
$\therefore$ " " $= \$\frac{100 \times 125}{80} = \$156\frac{1}{4}$;
$\therefore$ he marks his goods at an advance of $56\frac{1}{4}$ %.

167. Income from investing $\$101\frac{1}{2} = \$6$;
" " " $\$17255 = \$\frac{17255 \times 6}{101\frac{1}{2}}$
$= \$1020.$
Income from investing \$85 $= \$5$;
$\therefore$ " " " $\$17255 = \$\frac{17255 \times 5}{85}$
$= \$1015$;
$\therefore$ total income $= \$1020 + \$1015 = \$2035.$

168. Time required for 15 men working 9 hours a day to finish the work
$= 16$ days;
$\therefore$ to do the work in 1 day 1 man must work $(15 \times 16 \times 9)$ hrs.;
$\therefore$ to do it in 12 days 18 men must work $\frac{15 \times 16 \times 9}{12 \times 18}$ hrs.
$= 10$ hrs.

169. Increase of shorter in 100 yr. $= 3.014$ in.;
$\therefore$ " " 125 yr. $= \frac{125 \times 3.014}{100}$ in.
$= 3{\cdot}7675$;

hence the longer has to increase (3·7675 − 1·02) in. or 2·7475 in.

Increase in 125 yr. $= 2{\cdot}7475$ in.;

$\therefore$ " " 100 yr. $= \frac{100 \times 2{\cdot}7475}{125}$

$= 2.198$ in.

170. B walks at the rate of $\frac{50}{12}$ mi., or $4\frac{1}{6}$ mi. per hr.;

$\therefore$ B walks 20 miles in $(20 \div 4\frac{1}{6})$ hr. $= 4\frac{4}{5}$ hr.;

$\therefore$ A walks 20 miles in 1 hr. $+ 4\frac{4}{5}$ hr. $= 5\frac{4}{5}$ hr.;

$\therefore$ A walks 50 miles in $\frac{50 \times 5\frac{4}{5}}{20}$ hr. $= 14\frac{1}{2}$ hr.;

$\therefore$ A reaches London at 6 hr. 30 min. P.M.

171.

		10747·4689(103·67	
		1	
	203	747	
		609	
	2066	13846	
		12396	
	20727	145089	
		145089	
	5		189119224
			125
15	7	7500	64119
		1099	
		8599	60193
		49	
171	4	974700	3926224
		6856	
		981556	3926224

172. Time to read $(220\times28\times12)$ words $= 5\frac{1}{2}$ hr.;

$\therefore$ " " $(400\times36\times14)$ "

$= \frac{400\times36\times14\times5\frac{1}{2}}{220\times28\times12}$ hr.

$= 15$ hr.

173. Distance the train goes in 60×60 sec.

$= 20\times1760$ yd.;

$\therefore$ " " " 18 sec.

$= \frac{18\times20\times1760}{60\times60}$ yd.

$= 176$ yd.;

$\therefore$ length of bridge $= (176-120)$ yd.

$= 56$ yd.

174. 20 men $=$ 30 women, and 50 children $=$ 30 women;

$\therefore$ sum received by $(30+40+30)$ women

$= \$600$ for 1 week's work,

and sum received by 1 woman $= \$\frac{600}{100}$ " "

$= \$6$.

175. The first strikes the 7th stroke after $\frac{6\times30}{11}$ sec.;

the second strikes the 7th stroke after $\frac{6\times25}{11}$ sec.;

$\therefore$ the difference $= \frac{6\times10}{11}$ sec. $= 1\frac{1}{11}$ min.

176. Money received $= \$(43\times11\frac{1}{2})$;

$\therefore$ annual income $= \$\frac{43\times11\frac{1}{2}\times6}{12\times8}$

$= \$23.17\frac{1}{2}$.

177. Number of minutes between 9 a. m. Tuesday and 11 a. m. Wednesday $= 1560$.

Number of minutes between 9 a. m. Tuesday and 9 p. m. Wednesday $= 2160$.

The slow clock goes 1550 min. while the fast one goes 1560 min.;

$\therefore$ the slow clock goes 2160 min., while the fast one goes $\frac{2160\times1560}{1550}$ min., or $2173\frac{29}{31}$ min.;

$\therefore$ it must be put back $(2173\frac{29}{31} - 2160)$ min., or $13\frac{29}{31}$ min.

178. $\frac{17}{40} + \frac{8}{19}$ of $\frac{23}{40} = \frac{507}{760}$;

$\therefore$ $\frac{253}{760}$ of the ore $= 506$ tons;

$\therefore$ the ore $= \frac{760\times506}{253}$ tons

$= 1520$ tons.

179. The net annual increase is 1 in 60, and, hence the population of each year $= \frac{61}{60}$ of the population of the preceding year;

$\therefore$ population at end of 5 years $= (\frac{61}{60})^5$ of 10000000

$= 10861578$, nearly.

180. Length of side $= \frac{945\times1344}{1134}$ yd.

$= 1120$ yd.

Area of each $= \frac{945\times1344}{4840}$ acres.

$= 262\frac{50}{121}$ acres.

181. $\cdot0416 = \frac{10}{240}$ and $\cdot0375 = \frac{9}{240}$;

$\therefore$ $\frac{10}{240}$ of number of inmates at first $= \frac{9}{240}$ of (number of inmates at first + 40),

and $\frac{1}{240}$ of number of inmates at first $= \frac{9}{240}$ of 40;

$\therefore$ number of inmates at first $= 240\times\frac{9}{240}$ of 40

$= 360$;

$\therefore$ number of masters $= \frac{10}{240}$ of 360

$= 15$,

and number of boys $= 360-15$

$= 345$.

182. The shares are as 1, 3, 6 and 10.

$1+3+6+10 = 20$.

A's share $= \frac{1}{20}$ of \$350 $=$ \$17.50.

B's " $= 3 \times \$17.50 = \$52.50.$
C's " $= 6 \times \$17.50 = \$105.$
D's " $= 10 \times \$17.50 = \$175.$

183. What cost \$100 I sell for \$92½, and should sell for \$112½ to gain 12½ %;

$\therefore$ selling price of \$3700 $= \$\frac{3700 \times 112\frac{1}{2}}{92\frac{1}{2}}$
$= \$4500.$

184. Interest $= \$\frac{1265 \times 73 \times 6}{365 \times 100} = \$15.18.$

Discount $= \$\left(\frac{1265 \times 1\frac{1}{5}}{101\frac{1}{5}}\right) = \$15;$

$\therefore$ difference $= 18$ cents.

185. On \$100 outlay he should get \$160, but receives only three-eighths of \$160, that is, \$60. Thus he loses 40 per cent.

186. $\frac{98\frac{1}{2}}{100}$ of part of income over \$400 $= \$1024.40;$
$\therefore$ " " \$400
$= \$\frac{100 \times 1024.40}{98\frac{1}{2}}$
$= \$1040;$
$\therefore$ the gross income $= \$1440.$
Sum invested for income of \$6 $= \$101\frac{1}{2};$
$\therefore$ " " " \$1440 $= \$\frac{1440 \times 101\frac{1}{2}}{7}$
$= \$24360.$

187. His gross receipts are decreased 35 % by the fall in flour, and 5 % by the lowering of trade expenses, and, therefore 40 % in all.

Hence he can lower the 15 ct. loaf 40 %, that is by $\frac{2}{5}$ of 15 ct. or 6 ct.

188. $\frac{2}{7}$ of profits for 28 mos. $= \$7890.50;$
$\therefore$ total profits for 12 mos. $= \frac{7 \times 12 \times 7890.50}{2 \times 28}$
$= \$11835.75$

189. Cost price $= \frac{100}{112\frac{1}{2}}$ of $\$3.82\frac{1}{2} = \$3.40$.

190. Area of whole rectangle $= (72 \times 45)$ sq. yd.
$= 3240$ sq. yd.
Area of grass plots $= (4 \times 27 \times 13\frac{1}{2})$ sq. ft.
$= 162$ sq. yd. ;
$\therefore$ area to be gravelled $= (3240 - 162 - 36)$ sq. yd.
$= 3042$ sq. yd. ;
$\therefore$ cost $= \frac{3042 \times 8}{3}$ cts. $= \$81.12$.
Depth of pond $= \frac{252}{36}$ yd. $= 7$ yd.

191. $$\frac{\frac{11 \times 2 \times 18}{2 \times 9 \times 7} - 1 \div \frac{7}{10}}{1 - \frac{3}{14} \text{ of } \left\{\frac{1}{2} + \frac{1}{2} \text{ of } \frac{1}{3}\right\}} = \frac{\frac{22}{7} - \frac{10}{7}}{1 - \frac{3}{14} \text{of } \left\{\frac{1}{2} + \frac{1}{6}\right\}}$$
$$= \frac{\frac{12}{7}}{1 - \frac{1}{7}} = 2.$$

192. 9 boys are equivalent to 6 men ;
since 12 men do $\frac{3}{4}$ of the work in $6\frac{1}{2}$ hrs. ;
$\therefore$ 1 man does $\frac{1}{4}$ of the work in $\frac{12 \times 13}{3 \times 2}$ hrs. ;
$\therefore$ 17 men do $\frac{1}{4}$ of the work in $\frac{12 \times 13}{17 \times 3 \times 2}$ hrs.,
or $1\frac{9}{16}$ hr.

193. Principal on which $\$14\frac{1}{4}$ is interest $= \$100$;
$\therefore$ " $\$1068.75$ "
$= \$\frac{1068.75 \times 100}{14\frac{1}{4}}$
$= \$7500$.

194. Compound interest on $\$100 = \$8.16$.
Simple " " $= \$8$;
$\therefore$ sum on which $\$0{\cdot}16$ is difference $= \$100$;
$\therefore$ " " $\$6$ " $= \$\frac{6 \times 100}{.16}$
$= \$3750$.

195. The ratio of costs is as $2 \times 9 \times 25 \times 12$ to $1 \times 8 \times 18 \times 10\frac{1}{2}$;

$\therefore$ the cost of second vessel $= \frac{8\times18\times10\frac{1}{2}}{2\times9\times25\times12}$ of \$30000
$= \$8400.$

196. $\frac{99}{100}$ of a child's share $= \frac{2\times97}{100}$ of a brother's share;

$\therefore$ a brother's share $= \frac{99}{194}$ of a child's share;

hence 5 times a child's share $+ \frac{3\times99}{194}$ times a child's share $= \$12670$;

or, $\frac{1267}{194}$ times a child's share $= \$12670$;

$\therefore$ a child's share $= \$1940$, and a brother's share $= \$990$;

and, when the legacy duty has been taken away, each child will receive \$1920.60, and each brother \$960.30.

197. Interest on B's debt to $A = \$(5\times3\frac{1}{4}\times4) = \$65$.
P. W. of B's claim on $A = \frac{100}{103}$ of \$360.50 $= \$350$;
$\therefore$ B has to pay $\$565 - \$350 = \$215$.

198. £1 16*s*. 8*d*. $= 440d$.
Buying price per lb. $= \frac{440}{112}d. = \frac{55}{14}d.$
Gain on an outlay of $\frac{55}{14}d. = (\frac{9}{2} - \frac{55}{14})d.$
$= \frac{4}{7}d;$
$\therefore$ " " $100d. = \frac{100\times\frac{4}{7}}{\frac{55}{14}}d.$
$= 14\frac{6}{11}d.$

199. 20 %, or $\frac{1}{5}$ of the wheat grown in the country
$= 10000000$ quarters;
$\therefore$ wheat grown $= (5\times10000000)$ quarters
$= 50000000$ qrs.

200. Rate *with* the stream $= \frac{18}{4}$ mi. $= 4\frac{1}{2}$ mi. per hr.
Rate *against* " $= \frac{18}{12}$ mi. $= 1\frac{1}{2}$ mi. " ;
$\therefore$ rate in still water $= \frac{6}{2}$ mi. $= 3$ mi. " ;
$\therefore$ rate of stream $= 4\frac{1}{2}$ mi. $- 3$ mi. $= 1\frac{1}{2}$ mi. " .

201. $\dfrac{1}{1+\dfrac{1}{2+\dfrac{1}{3+\frac{5}{21}}}} + \dfrac{1}{5+\dfrac{1}{4+\dfrac{1}{3+\frac{1}{3}}}}$

$= \dfrac{1}{1+\dfrac{1}{2+\frac{21}{68}}} + \dfrac{1}{5+\dfrac{1}{4+\frac{3}{10}}} = \dfrac{1}{1+\frac{68}{157}} + \dfrac{1}{5+\frac{10}{43}}$

$= \frac{157}{225} + \frac{43}{225} = \frac{200}{225} = \frac{8}{9}$.

202. Part done by A, B, and C, daily $= \frac{1}{4}$.

" " A and B " $= \frac{1}{6}$.

" " B and C " $= \frac{1}{8}$;

$\therefore$ " " A " $= \frac{1}{4} - \frac{1}{8}$

$= \frac{1}{8}$.

" " C " $= \frac{1}{4} - \frac{1}{6}$

$= \frac{1}{12}$;

$\therefore$ " " A and C " $= \frac{1}{8} + \frac{1}{12}$

$= \frac{5}{24}$;

$\therefore$ A and C can do the work in $\frac{24}{5}$ da., or $4\frac{4}{5}$ days.

203. Cost price $= \frac{100}{92}$ of \$38.25

$= \$\frac{3825}{92}$;

$\therefore$ gain $= \$(57 - \frac{3825}{92})$;

$\therefore$ gain per cent. $= \$\dfrac{100 \times (57 - \frac{3825}{92})}{\frac{3825}{92}}$

$= \$\dfrac{100 \times (5244 - 3825)}{3825}$

$= \$37\frac{5}{51}$.

204. 1 mile $= (1760 \times 36)$ in. $= \frac{63360}{39.371}$ metres

$= 1609.306 \ldots$ metres

205. P. W. of \$2.05 $= \frac{100}{1024}$ of \$2.05 $= \$2$.

206. The amount of \$1 $= (1{\cdot}02)^4 = \$1.08243\ldots$.

$\therefore$ interest of \$100 $= 100 \times \$.08243\ldots = \$8.243\ldots$.

207. Money realized by sale $= \$\frac{9790\times98}{100}$.

Income from M. B. stock $= \$\frac{9790\times98\times12}{100\times178}$

$= \$646.80.$

Original income $= \$\frac{9790\times6}{100}$

$= \$587.40$;

$\therefore$ difference $= \$59.40.$

208. Cost price of 1 quarter of mixture

$= \frac{100}{125}$ of $57s.\ 6d.$

$= 46s.$

Sum gained on each quarter of the cheap kind is $7s.$

Sum lost on each quarter of the dear kind is $2s.$;

$\therefore$ they must be mixed in the rates of 2 : 7.

209. Cubic content of block $= \frac{8\times2\times3}{1728}$ c. ft.

$= \frac{1}{36}$ c. ft.

Weight of $\frac{1}{36}$ c. ft. of water $= \frac{1}{36}$ of $\frac{1000}{16}$ lb.;

$\therefore$ " $\frac{1}{36}$ c. ft. of gold $= 19.26\times\frac{1}{36}$ of $\frac{1000}{16}$ lb.

$= 33$ lb. 7 oz.

210. Content of cistern $= (1000+8)$ c. in.

$= 1008$ c. in.

Area of base $= (\frac{121}{81}-\frac{100}{81})$ sq. ft.

$= \frac{21}{81}$ sq. ft.

$= \frac{21\times144}{81}$ sq. in.;

$\therefore$ depth $= (1008\div\frac{21\times144}{81})$ in.

$= 27$ in.

211. $\frac{857142}{999999} = \frac{6}{7}$.

$\frac{6}{7}$ of £10 14$s.$ 1$d.$ = £9 3$s.$ 6$d.$

Again, ·85714 of £10 14$s.$ 1$d.$ = ·85714 of 2569$d.$

$= 2201{\cdot}99266d.$

£9 3$s.$ 6$d.$ $= 2202d.$

Difference $= {\cdot}00734d.$, which is less than $\frac{1}{1272}d.$

212. \$400 for 3 mos. gives the same interest as \$100 for 1 year, and since the rate is double that on the \$327, the interest at the end of the year will be the same as the interest for a year on \$327 + \$200 at the smaller rate;

$\therefore$ interest on \$527 for 1 yr. = \$26.35;

$\therefore$ " \$100 " $= \$\frac{100 \times 26.35}{527} = \$5$;

$\therefore$ the rates are 5 % and 10 %.

213. Cost of a gallon of mixture $= (3 \times \frac{109}{120})s. = 2\frac{1}{2}s.$

But $2\frac{1}{2}s.$ is $\frac{3}{8}$ of $4s.$;

$\therefore$ $\frac{3}{8}$ of the mixture is water

i. e. there are 3 pints of water in each gallon.

214. Interest on \$550 for 9 mos. = \$16.50;

$\therefore$ " \$100 " 12 " $= \$\frac{100 \times 12 \times 16.50}{550 \times 9}$

$= \$4.$

215. The broker first offered $\frac{9}{10}$ of the value;

then $\frac{9}{10}$ of the value + \$379.75 = $\frac{105\frac{1}{2}}{100}$ of the value;

$\therefore$ \$379.75 $= (\frac{211}{200} - \frac{9}{10})$ of the value $= \frac{31}{200}$ of the value;

$\therefore$ the value $= \frac{200}{31}$ of \$379.75 = \$2450.

216. Asking price $= \frac{125}{100}$ of cost price;

$\therefore$ selling price $= \frac{85}{100}$ of $\frac{125}{100}$ of cost price

$= \frac{106\frac{1}{4}}{100}$ of cost price;

$\therefore$ $\frac{6\frac{1}{4}}{100}$ of cost price = \$5.75;

$\therefore$ cost price $= \$\frac{100 \times 5.75}{6\frac{1}{4}} = \$92$;

and asking price $= \frac{125}{100}$ of \$92 = \$115.

217. 15 masons build 200 sq. yd. in 60 hours;

$\therefore$ 1 mason builds 1 sq. yd. in $\frac{15 \times 60}{200}$ hrs.;

$\therefore$ 7 masons build 420 sq. yd. in $\frac{420 \times 15 \times 60}{7 \times 200}$ hrs.;

$\therefore$ they take 270 hrs., or 30 days.

218. The average dividend $= \$\frac{.75+.60}{2} = \$\cdot 67\frac{1}{2}$.

His debts are $\frac{100}{67\frac{1}{2}}$ of \$2700 = \$4000.

219. Toll on 240 hhd. = 2 hhd. + \$90.

" 150 hhd. = 2 hhd. − \$30;

∴ " 90 hhd. = \$120;

∴ " 150 hhd. $= \$\frac{150\times120}{90} = \$200$;

∴ value of 2 hhd. = \$200 + \$30 = \$230;

∴ " 1 hhd. $= \$\frac{230}{2} = \$115$.

220. Area of walls $= (80\times6)$ sq. yd. $+ 2(80\times5)$ sq. yd.

$= 1280$ sq. yd.

Deduction $= (6\times8\times3)$ sq. ft. $= 16$ sq. yd.;

∴ number of pictures $= \frac{1264\times9}{8\times3} = 474$.

221. $\frac{\frac{32}{9}-\frac{165}{90}}{\frac{88}{9}-\frac{58}{9}} \times \frac{1}{71} \times \frac{\frac{213}{99}}{\frac{31}{10}\times\frac{100}{999}}$

$= \frac{320-165}{880-580} \times \frac{1}{71} \times \frac{21300}{3100}$

$= \frac{155}{300} \times \frac{3}{31} = \frac{1}{20} = \cdot 05$.

222. 15376·248001 (124·001

1

22 | 53
44

244 | 976
976 $\quad \surd\frac{31.36}{30.69} = \surd\frac{4.48}{5.67} = \surd\frac{.64}{.81} = \frac{.8}{.9} = \frac{8}{9}$.

248001 | 248001
248001

223. $\frac{2}{3} + \frac{1}{9} = \frac{7}{9}$;

∴ $\frac{2}{9}$ of the army = 2000 men;

∴ whole army $= \frac{9\times2000}{2}$ men = 9000 men.

224. The interest on \$2000 for 3 mos. = \$37.50.;

∴ at the end of 2 years the second would have re-

ceived \$19000 + 7 × \$37.50 + 6 × \$37.50 + 5 × \$37.50 + 4 × \$37.50 + 3 × \$37.50 + 2 × \$37.50 + \$37.50, or \$20050, which is more than the first tender by \$50.

225. An income of \$5 is got from an investment of \$91¼;

∴ " \$450 is got from an investment of $\$\frac{450\times 91\frac{1}{4}}{5}$;

∴ $\frac{90}{100}$ of sum left $= \$\frac{450\times 91\frac{1}{4}}{5}$;

∴ sum left $= \$\frac{100\times 450\times 91\frac{1}{4}}{90\times 5} = \$9125.$

226. Part done by 2 men and 4 boys hourly $= \frac{1}{2}$.

" " 2 " 1 boy " $= \frac{1}{3}$;

∴ " " 3 boys " $= \frac{1}{2}-\frac{1}{3}$

$= \frac{1}{6}$;

∴ " " 1 boy " $= \frac{1}{18}$;

hence 1 boy would do the whole in 18 hr.

Part done by 2 men hourly $= \frac{1}{3}-\frac{1}{18}$

$= \frac{5}{18}$;

∴ " " 1 man " $= \frac{5}{36}$;

hence 1 man would do the whole in $\frac{36}{5}$ hr., or $7\frac{1}{5}$ hr.

Part done by 1 man and 1 boy hourly $= \frac{5}{36}+\frac{1}{18}$

$= \frac{7}{36}$;

hence 1 man and 1 boy would do the whole in $\frac{36}{7}$ hr., or $5\frac{1}{7}$ hr.

227. Interest on \$15840 $= \$(15840\times\frac{3}{12}\times\frac{8}{100})$

$= \$316.80.$

Interest $= (\frac{15}{12}\times\frac{74}{100})$ of the sum

$= \frac{3}{32}$ of the sum;

∴ discount $= \frac{3}{35}$ of \$3696

$= \$316.80.$

228. See solution of Ex. 168, page 144.

229. \$20 = interest on \$340. (Art. 181);
$\therefore$ \$360 = " $\$\frac{360\times340}{20}$
= \$6120.

230. $(2\times\text{breadth})\times\text{breadth}\times\frac{\text{breadth}}{2} = 4096$ c. ft.;
$\therefore$ cube of breadth = 4096 c. ft.,
and breadth = $\sqrt[3]{4096}$ ft.
= 16 ft.;
$\therefore$ length = 32 ft. and height = 8 ft.

231. 1 lb. tea = $\frac{50\times84}{70}$ lemons
= $(5\times12)d$.
= $5s$.

232. Number of killed and wounded
= $\frac{1}{8}$ of $\frac{2}{7}$ of $\frac{1}{3}$ of army
= $\frac{1}{84}$ of army;
$\therefore$ $\frac{1}{84}$ of army = 500 men;
$\therefore$ army = 84×500 men
= 42000 men.

233. Cash price in notes = $\frac{19}{20}$ of \$135 = \$128.25.
" " gold = $\frac{100}{120}$ of \$128.25
= \$106.87½.
Change to be received in gold = \$(135 − 106.87½)
= \$28.12½;
$\therefore$ " " " notes = $\frac{6}{5}$ of \$28.12½
= \$33.75.

234. Interest = interest on debt.
Discount = interest on present worth;
$\therefore$ interest − discount = interest on (debt − P. W.)
= interest on discount.
(See Art. 181.)

235. Cost price $= 20 \times 16 \times 55$ cents $= \$176$.

No. of Troy oz. bought $= \frac{20 \times 7000}{20 \times 24}$.

Selling price $= \frac{20 \times 7000 \times 60}{20 \times 24}$ cts. $= \$175$;

$\therefore$ loss $= \$1$.

236. \$6 is got from investing $\$91\frac{1}{2}$;

$\therefore$ \$320 " " $\$\frac{320 \times 91\frac{1}{2}}{6}$.

Income from \$80 $= \$5$;

$\therefore$ " " $\$\frac{320 \times 91\frac{1}{2}}{6} = \$\frac{320 \times 91\frac{1}{2} \times 5}{6 \times 80}$

$= \$305$.

237. P. W. of debt $= \$\frac{28 \times 100}{110} = \$25\frac{5}{11}$;

$\therefore$ difference $= \$(25\frac{5}{11} - 25.20) = \$\frac{14}{55}$.

238. $\frac{105}{300}$ of his property + $\frac{103}{600}$ of his property + $\frac{102}{200}$ of his property = £6190;

$\therefore$ $\frac{619}{600}$ of his property = £6190;

$\therefore$ " $= £\frac{600 \times 6190}{619}$

$= £6000$.

239. Sum invested $= \$\frac{3681 \times 100}{102\frac{1}{2}} = \$3600$.

No. of bbl. bought $= \frac{3600}{7.50} = 480$.

Total selling price $= \frac{120}{100}$ of $\$(3681 + 119) = \$4560$;

$\therefore$ selling price of 1 bbl. $= \$\frac{4560}{480} = \$9.50$.

240. Content of wall $= (60 \times 20 \times 4)$ cub. ft.

Space occupied by bricks $= \frac{931}{100}$ of 4800 cub. ft.

$= (375 \times 12)$ cub. ft.;

$\therefore$ number of bricks $= \frac{375 \times 12 \times 17'28}{9 \times 4\frac{1}{2} \times 4} = 48000$.

241. $\frac{1}{4} \times \frac{1}{14} \times \frac{7}{7}$ of $168s. = \frac{42}{14}s. = 3s.$

242. A, B, and C together do $\frac{1}{2}$ of $(\frac{1}{4} + \frac{1}{6} + \frac{1}{8})$ in 1 day;

$\therefore$ they do $\frac{13}{48}$ in one day;

$\therefore$ they do the whole in $\frac{48}{13}$ days $= 3\frac{9}{13}$ days.

243. For every 3 days of the time he earned $\$(2 \times 1.50)$ and paid 60 cents, or his net earning was \$2.40.

Time he took to earn \$2.40 = 3 days ;

$\therefore$ " " " \$72 $= \frac{72 \times 3}{2.40}$ days = 90 days.

244. $\frac{1}{6} + \frac{1}{7} + \frac{1}{8} = \frac{28+24+21}{168} = \frac{73}{168}$;

$\therefore$ 1st gets $\frac{\frac{1}{6}}{\frac{73}{168}}$ of \$146000 = \$56000.

2nd gets $\frac{\frac{1}{7}}{\frac{73}{168}}$ of \$146000 = \$48000.

3rd gets $\frac{\frac{1}{8}}{\frac{73}{168}}$ of \$146000 = \$42000.

245. Interest on \$200 for 3 mo. = \$10.

" \$200 for 1 yr. = \$40.

Discount off \$240 for 1 yr. = \$40 ;

$\therefore$ " \$210 for 1 yr. $= \$\frac{210 \times 40}{240} = \$35$.

246. Since £3 17*s.* 10½*d.* = 1869 half-pence,

and 1 sovereign = 480 half-pence,

the least number of sovereigns will be the L. C. M. of 1869 and 480 half-pence.

L. C. M. of 1869 and 480 $= 3 \times 623 \times 160$.

But $(3 \times 623 \times 160)$ half-pence

= 623 sovereigns,

and 623 sovereigns weigh 160 oz.

247. Investment to give \$7 dividend = \$175.

Investment to give \$445.50 dividend

$= \$\frac{445.50 \times 175}{7}$

= \$11137.50 ;

$\therefore$ selling price of flour = $\frac{100}{99}$ of \$11137.50;

$\therefore$ number of bbls. $= \frac{11137.50}{7.50 \times 99}$

= 1500.

248. 1 lb. Troy $= \frac{144}{175}$ lb. Avoir. (Art. 157).

Weight of rings $= \frac{1050 \times 28}{12 \times 20 \times 24}$ lb. Troy

$= \frac{144 \times 1050 \times 28}{175 \times 12 \times 20 \times 24}$ lb. Avoir.

$= \frac{21}{5}$ lb. Avoir.

Weight of rings and box $= (\frac{21}{5} + 3\frac{1}{2})$ lb.

$= 7\frac{7}{10}$ lb.

Cost of carrying 1 ton, 1 mi. $= 5s.$;

$\therefore$ " $7\frac{7}{10}$ lb., 144 mi. $= \frac{7\frac{7}{10} \times 144 \times 5}{2240} s.$

$= \frac{99}{40} s.$

Value of rings $= 1050 \times 22s.$;

$\therefore$ cost of insurance $= \frac{1}{800} \times 1050 \times 22s.$

$= \frac{1155}{40} s.$;

$\therefore$ total cost $= \frac{99 + 1155}{40} s.$

$= £1\ 11s.\ 4{\cdot}2d.$

249. Interest for 1st year $= \$250$;

" 2nd " $= \$275$;

" 3rd " $= \$302.5$;

" 4th " $= \$332.75$;

" 5th " $= \$366.025$;

$\therefore$ the sum of these is $\$1526.275$; so that the interest to be gained after the 5th year is $\$201.31\frac{3}{8}$, but the interest for the 6th year $= \$402.62\frac{3}{4}$.

$\therefore$ $5\frac{1}{2}$ years is the time required.

250. Length $= \frac{3}{2}$ of breadth, and height $= \frac{2}{3}$ of breadth;

$\therefore$ $\frac{3}{2}$ of breadth $\times$ breadth $\times \frac{2}{3}$ of breadth $= 5832$ c. ft.,

and cube of breadth $= 5832$ c. ft.;

$\therefore$ breadth $= \sqrt[3]{5832}$ ft.

$= 18$ ft.

Length $= 27$ ft.

Height $= 12$ ft.

251. $\begin{array}{r} 121711 \\ (32)(8)(56) \\ \hline 973688 \quad = 800 \text{ times the multiplicand.} \\ 6815816 \quad = 7 \text{ times } 8 \text{ times } 121711. \\ 3894752 \quad = 40 \text{ times } 800 \text{ times } 121711. \\ \hline 3998936616. \end{array}$

252. $\dfrac{\frac{28}{10}\times\frac{225}{99}}{\frac{125}{990}} + \dfrac{\frac{40}{9}-\frac{255}{90}}{\frac{15}{9}+\frac{2627}{999}} \times \frac{2040}{225}$

$= \frac{28\times225}{1125} + \frac{145\times204\times111}{4292\times225} = \frac{28}{5} + \frac{29\times51\times111}{1073\times45}$

$= \frac{28}{5} + \frac{153}{45} = 9.$

253. Commission on \$2480 = \$21.70;

$\therefore$ " " \$100 $= \$\frac{100\times21.70}{2480}$

$= \$\frac{7}{8}$;

$\therefore$ his commission was at the rate of $\frac{7}{8}$ %.

254. Income from \$92 realized = \$6.

" \$25760 " $= \$\frac{25760\times6}{92}$

$= \$1680.$

" investing \$45½ = \$3¼.

" " \$25760 $= \$\frac{25760\times3\frac{1}{4}}{45\frac{1}{2}}$

$= \$1840$;

$\therefore$ his income is increased \$160.

255. P. W. of \$348 $= \$\frac{348\times100}{101\frac{1}{2}} = \frac{34800\times2}{203}$

P. W. of \$292 $= \$\frac{292\times100}{100\frac{1}{2}} = \frac{29200\times8}{803}$;

$\therefore$ total gain $= \$\left(\frac{34800\times2}{203} - \frac{29200\times8}{803}\right)$;

$\therefore$ gain per cent $= \dfrac{100\left(\frac{34800\times2}{203} - \frac{29200\times8}{803}\right)}{\frac{29200\times8}{803}}$

$$= \frac{34800\times2\times803-29200\times8\times203}{\frac{203\times803}{29\times8}}$$

$$= \frac{34800\times2\times803-29200\times8\times203}{203\times292\times8}$$

$$= \frac{34800\times2\times11-400\times8\times233}{203\times4\times8}$$

$$= \frac{2175\times11-100\times203}{203}$$

$$= \frac{3625}{203} = 17\frac{6}{7}.$$

256. Since there is a difference of half a day in the time of completion, according as the boy or man commences first, the man must do twice as much work each day as the boy

Part of work done in one day by both $= (\frac{1}{13} + \frac{2}{13})$;

$\therefore$ they will finish the work in $\frac{13}{3}$ days,

or $4\frac{1}{3}$ days.

257. Share of 1st $= \frac{30}{100}$ of \$300 $=$ \$90.

" 2nd $= \frac{30}{100}$ of \$100 $=$ \$30.

But as the machine works the same length of time for each and earns \$120, in all, or \$60 for each, therefore, the latter must give the former the difference between \$60 and \$30, or \$30.

258. Since B gets \$2750 at the end of two years, he receives $\frac{100}{110}$ of \$2750, or \$2500;

A calculates \$2500 to be the P. W. of \$2725, that is, that the interest on \$2500 for 2 years is \$225;

$\therefore$ " \$100 " 1 yr. $= \$\frac{100\times225}{2500\times2}$

$= \$4\frac{1}{2}$.

259. Time 36 men dig $(72\times18\times12)$ c. yd.

$= (16\times8)$ hr.;

$\therefore$ time 1 man digs 1 c. yd $= \frac{36\times16\times8}{72\times18\times12}$ hr.

$= \frac{8}{9\times3}$ hr.;

$\therefore$ time 32 men dig $(64 \times 27 \times 18)$ c. yd $= \frac{64 \times 27 \times 18 \times 8}{32 \times 9 \times 3}$ hr.

$= \frac{2 \times 3 \times 6 \times 8}{12}$ da.

$= 24$ da.

260. P. W. of bill $= £\frac{180 \times 100}{120} = £150$.

The wine and picture are charged at $£(21 + 19)$, or £40;

$\therefore$ he pays in cash $£(150 - 40)$, or £110;

$\therefore$ the cash price of the bill to the usurer is $£(110 + 10)$, or £120;

$\therefore$ the interest on £120 for 4 mo. $=$ £60;

$\therefore$ " " £100 for 12 mo. $= £\frac{100 \times 12 \times 60}{120 \times 4}$

$=$ £150.

261. $\frac{1}{10} + \frac{1}{20} + \frac{1}{30} + \frac{1}{40} + \frac{1}{50} + \frac{1}{60} = \frac{147}{600} = \frac{49}{200}$;

$\therefore$ length of $\frac{151}{200}$ of rod $= 302$ in.;

$\therefore$ length of rod $= \frac{200 \times 302}{151}$ in.

$= 400$ in.

262. \$0.75 is 9 mo. interest on \$20;

$\therefore$ \$20.75 " " " $\$\frac{20.75 \times 20}{.75}$,

or on $\$553\frac{1}{3}$.

Again, interest on \$20 for 9 mo. $=$ \$0.75;

$\therefore$ " \$100 for 12 mo. $= \$\frac{100 \times 12 \times .75}{20 \times 9}$

$=$ \$5.

263. Distance the first goes $= \frac{10 \times 3}{60}$ mi. $= \frac{3}{6}$ mi.

" " second " $= \frac{10 \times 4}{60}$ mi. $= \frac{4}{6}$ mi.;

$\therefore$ length of walk $= (\frac{3}{6} + \frac{4}{6})$ mi. $= 1\frac{1}{6}$ mi.

264. A runs 100 yards while B runs 96 yards;

A runs 100 yards while C runs 95 yards;

$\therefore$ B runs 96 yards while C runs 95 yards;

hence B, giving C 1 yard start, will overtake him at the end of 96 yards, and will therefore beat him in a hundred yards' race.

265. The selling price $= \frac{105}{100}$ of cost price;
$\therefore \frac{110}{100}$ of $\frac{95}{100}$ of cost price $= \frac{105}{100}$ of cost price $- \$1$;
$\therefore \left(\frac{105}{100} - \frac{1045}{1000}\right)$ of cost price $= \$1$;
$\therefore \frac{5}{1000}$ of cost price $= \$1$;
and cost price $= \$200$.

266. Compound interest of \$1 $= \{(1.05)^2 - 1\}$
$= \$0.1025$.
Simple " \$1 $= \$0.10$;
$\therefore \$.0025$ = the difference on \$1;
$\therefore \$3$ = " $\$\frac{3 \times 1}{.0025} = \$1200$.

267. Cost price $= \frac{100}{107}$ of \$69.55 $= \$65$;
$\therefore$ loss is \$65 − \$61.75 $= \$3.25$;
$\therefore$ loss per cent $= \frac{100 \times 3.25}{65} = 5$.

268. Cost of £360 $= \$1736.10$
" 1 $= \$4.82\frac{1}{4}$;
But this includes the commission at $\frac{1}{4}\%$;
hence the course of exchange $= \frac{400}{401}$ of $\$4.82\frac{1}{4}$
$= \$4.81$ nearly.

269. 3 mos. $= \frac{1}{4}$ year, and $\frac{1}{4}$ of $8\% = 2\%$.
Discount at 2% $= \frac{2}{102}$ of bill.
Interest " $= \frac{2}{100}$ "
Hence, $\left(\frac{2}{100} - \frac{2}{102}\right)$ of bill $= \$16$;
or $\frac{4}{10200}$ " $= \$16$;
$\therefore$ the bill $= \$\frac{10200 \times 16}{4}$
$= \$40800$.

270. Cost of 30 lb. $= \$(18 \times .30 + 12 \times .05) = \$6$.
Selling price $= \frac{125}{100}$ of \$6 $= \$7.50$;
$\therefore$ " of 1 lb. $= \$\frac{7.50}{30} = 25$ cents.

271.

		3·14159) 1·000000000 (·31831.....
		942477
11	7·00	575230
	·63	314159
2		2610710
	·318181818	2513272
	·000039772	974380
	·318221590	942477
		319030
		314159

∴ the difference lies between ·0001 and ·0002.

272. See solution of Ex. 4, paper II., page 71.

273. C's share $= \frac{6}{7}$ of D's share;
B's share $= \frac{4}{5}$ of $\frac{6}{7}$ of D's share $= \frac{24}{35}$ of D's share;
A's share $= \frac{2}{3}$ of $\frac{24}{35}$ of D's share $= \frac{16}{35}$ of D's share;
∴ $(1 + \frac{6}{7} + \frac{24}{35} + \frac{16}{35})$ of D's share $=$ \$21000;
∴ $\frac{105}{35}$ of D's share $=$ \$21000;
∴ D's share $=$ \$7000;
∴ C gets \$6000, B gets \$4800, A gets \$3200.

274. Amount of \$6.30 at end of 6 mo. $= \$\frac{6.30 \times 104}{100}$
$= \$6{\cdot}552.$
∴ buying at \$6.50 on 6 mo. is the more profitable,
or present value of \$6.50 $= \$\frac{6.50 \times 100}{104}$ $= \$6.25$;
∴ buying at \$6.50 on 6 mo. is the more profitable.

275. Income $= \$\frac{10.50 \times 100}{1\frac{1}{2}} = \$700.$
Tax on \$700 $= \$12.25$;
∴ " \$1 $= \$\frac{12.25}{700} = 1\frac{3}{4}$ cents.

276. Cost price of iron $= \$\frac{1260\times100}{120} = \$1050$;

$\therefore$ " $187\frac{1}{2}$ cwt. $= \$1050$;

$\therefore$ " 1 cwt. $= \$\frac{1050}{187\frac{1}{2}} = \$5.60$.

277. Sum invested in tea $= \$\frac{3060\times100}{102} = \$3000$.

Number of pounds bought $= \frac{3000}{.75} = 4000$.

Selling price $= \frac{125}{100}$ of $\$(3060+30+\frac{1}{300}$ of $3000)$
$= \$3875$;

$\therefore$ selling price of 1 lb. $= \$\frac{3875}{4000} = 96\frac{7}{8}$ cents.

278. Interest received each year $= \$2\frac{1}{2}+\$2\frac{1}{2}+$(interest on $\$2\frac{1}{2}$ for 6 mo.) $= \$5\frac{5}{80}$;

$\therefore$ on each \$100 he gains $\$(5\frac{5}{80}-3\frac{1}{2}) = \$1\frac{45}{80}$;

$\therefore$ to gain \$200 he must borrow $\$\frac{200\times100}{1\frac{45}{80}}$
$= \$12800$.

279. Cost of tea $= 250 \times 80$ cents $= \$200$.

Allowance for carriage $= \frac{21}{100}$ of $\$200 = \$5$;

$\therefore$ customer has to pay $\$(9.30 - 5) = \$4.30$.

280. Cubic content of the plate $= \frac{2}{8}$ c. in.;

$\therefore$ thickness of new surface $= \{\frac{2}{8}\div(7\times7\times9\times144)\}$ inches;

$= \frac{1}{8\times7\times7\times144}$ in. $= \frac{1}{56448}$ in.

281. After the sale to A he has left $\frac{92}{100}$, or $\frac{23}{25}$ of the flock;

after the sale to B he has left $\frac{23}{25}$ of the flock -90;

after the sale to C he has left

$\frac{193}{200}$ of $\{\frac{23}{25}$ of the flock $- 90\}$;

$\therefore$ $\frac{193}{200}$ of $\{\frac{23}{25}$ of the flock $- 90\} = 579$;

$\therefore$ $\frac{23}{25}$ of the flock $- 90 = \frac{200\times579}{193} = 600$;

$\therefore$ $\frac{23}{25}$ of the flock $= 690$;

$\therefore$ the whole flock $= \frac{25\times690}{23} = 750$.

282. $(1\text{st.})^2 \times (2\text{nd.})^2 \times (3\text{rd.})^2$

$= 176382 \times 279152 \times 215496;$

$\therefore$ 1st. $\times$ 2nd. $\times$ 3rd. $= \sqrt{(176382 \times 279152 \times 215496)}$

$= 2 \times 8 \times 9 \times 41 \times 73 \times 239;$

$\therefore$ 1st number $= \frac{2\times8\times9\times41\times73\times239}{215496} = 478.$

2nd " $= \frac{2\times8\times9\times41\times73\times239}{279152} = 369.$

3rd " $= \frac{2\times8\times9\times41\times73\times239}{176382} = 584.$

283. Sum of rates $= \frac{180+150}{3}$ ft. $= 110$ ft. per sec.

Difference of rates $= \frac{180+150}{15}$ ft. $= 22$ ft. " ;

$\therefore$ twice the rate of faster $= (110 + 22)$ ft. per sec. ;

$\therefore$ rate of faster $= \frac{60\times60\times66}{5280}$ mi. per hr.

$= 45$ mi. per hr.

Twice the rate of slower $= (110 - 22)$ ft. per sec. ;

$\therefore$ the rate of slower $= \frac{60\times60\times44}{5280}$ mi. per hr.

$= 30$ mi. per hr.

284. Cost of tea $= \frac{5}{6}$ of 72 cents $= 60$ cents ;

$\therefore$ gain per cent. $= \frac{100\times30}{60} = 50.$

285. Total cost $= \$(175 \times \cdot60 + 225 \times \cdot50) = \$217.50;$

$\therefore$ the selling price of the whole $= \frac{120}{100}$ of \$217.50

$= \$261.$

He sells 250 lb. for 250×55 cts. $= \$137.50;$

$\therefore$ " $(175 + 225 - 250)$ lb. for \$123.50 ;

$\therefore$ " he sells 1 lb. for $\$\frac{123.50}{150}$, or $82\frac{1}{3}$ cents.

286. Weight of nitre $= (\frac{75}{100} + \frac{77}{100})$ of 10 cwt.

$= 15\frac{1}{5}$ cwt.

" sulphur $= (\frac{10}{100} + \frac{9}{100})$ of 10 cwt.

$= 1\frac{9}{10}$ cwt.

" charcoal $= (\frac{15}{100} + \frac{14}{100})$ of 10 cwt.

$= 2\frac{9}{10}$ cwt.

287. Water admitted in 1 hr. $= 5 \times 3\frac{3}{4}$ t. $= 18\frac{3}{4}$ t. ;
$\therefore$ water gains $(18\frac{3}{4} - 12)$ t., or $6\frac{3}{4}$ t. in 1 hr.

Number of hours to gain 60 t. $= \dfrac{60}{6\frac{3}{4}} = 8\frac{8}{9}$ hr. ;

$\therefore$ rate of sailing $= \dfrac{40}{8\frac{8}{9}}$ mi. $= 4\frac{1}{2}$ m

288. $\frac{4}{100}$ of $\frac{4}{5}$ of the capital $=$ \$32000 ;
$\therefore$ $\frac{4}{100}$ of the whole " $= \$\dfrac{5 \times 32000}{4}$,
and " " $= \$\dfrac{100 \times 5 \times 32000}{4 \times 4}$
$=$ \$1000000.

$\frac{10}{100}$ of $\frac{1}{5}$ of capital $=$ \$20000 ;
$\therefore$ $\frac{52}{100}$ of the receipts $=$ \$(32000 + 20000),
and " $= \$\dfrac{100 \times 52000}{52}$
$=$ \$100000.

289. Interest $= \dfrac{\text{debt} \times 2\frac{1}{2} \times \text{rate}}{100}$;

discount $= \dfrac{\text{debt} \times 2\frac{1}{2} \times \text{rate}}{100 + 2\frac{1}{2} \times \text{rate}}$;

$\therefore$ $\dfrac{\text{interest}}{\text{discount}} = \dfrac{100 + 2\frac{1}{2} \times \text{rate}}{100}$, or, $\frac{87}{80} = 1 + \dfrac{\text{rate}}{40}$,

or, $\dfrac{\text{rate}}{40} = \frac{7}{80}$, or, rate $= 3\frac{1}{2}$.

290. $\dfrac{\text{Rate of boat} + \text{rate of stream}}{\text{Rate of boat} - \text{rate of stream}} = \frac{60}{55}$;

$\therefore$ 55 $\times$ rate of boat + 55 $\times$ rate of stream
$=$ 60 $\times$ rate of boat $-$ 60 $\times$ rate of stream ;

$\therefore$ 115 $\times$ rate of stream $=$ 5 $\times$ rate of boat ;

$\therefore$ 23 $\times$ rate of stream $=$ rate of boat.

291. $(\alpha)\ \dfrac{2+\dfrac{1}{3-\frac{6}{31}}}{\frac{13}{8}\div\frac{7\times29}{4\times2}} = \dfrac{2+\frac{31}{87}}{\frac{13}{7\times29}} = \dfrac{205\times7\times29}{13\times87} = 36\frac{31}{39}$;

$(\beta)\ \dfrac{6\frac{3}{4}+17\frac{2}{7}-7\frac{1}{4}}{3\frac{1}{5}+2\frac{1}{3}-4\frac{1}{10}} = \dfrac{16\frac{11}{14}}{1\frac{3}{5}} = \dfrac{235\times5}{14\times8} = 10\frac{55}{112}$.

292. 9 men and 15 women do $\frac{3}{8}$ of the work daily;
4 men and 14 children do $\frac{1}{6}$ " " ;
$\therefore$ 13 men, 15 women and 14 children do $(\frac{3}{8}+\frac{1}{6})$ of work daily, or $\frac{13}{24}$ of work;
$\therefore$ they do the work in $1\frac{11}{13}$ days.

293. Proceeds of sale $= £(100 \times 93\frac{1}{2}) = £9350$.
Income from 4 % $= £\frac{9350\times4}{102} = £366\ 13s.\ 1d.$
Original income $= £\frac{10000\times2}{100} = £300$;
$\therefore$ increase $= £66\ 13s.\ 4d.$

294. 5 men in $\frac{53}{25}$ hr. do $\frac{901}{1500}$ of work;
$\therefore$ 1 man in 1 hr. does $\frac{17}{300}$ " ;
$\therefore$ 3 men in 3 hr. do. $\frac{51}{100}$ " ;
hence 7 boys in 3 hr. do $\frac{49}{100}$ " ;
$\therefore$ 1 boy in 1 hr. does $\frac{7}{300}$ " ;
$\therefore$ 6 boys in 1 hr. do $\frac{7}{50}$ " .
Work to be done by 6 boys $= 1 - \frac{901}{1500} = \frac{599}{1500}$
Time for 6 boys to do $\frac{7}{50}$ $= 1$ hr.;
$\therefore$ " " " $\frac{599}{1500} = \dfrac{\frac{599}{1500}\times1}{\frac{7}{50}}$ hr.
$= 2{\cdot}8523809$ hr.

295. See solution of example 5, paper IV., page 165.

296. A does $\frac{1}{24}$ of work daily;
$\therefore$ he does $\frac{1}{48}$ of work in half a day;
$\therefore$ in two days A and B do $(1-\frac{5}{6}-\frac{1}{48})$ of work;
$\therefore$ in one day A and B do $\frac{7}{48}\div2 = \frac{7}{96}$ of work;
$\therefore$ in one day B does $\frac{7}{96}-\frac{1}{24} = \frac{3}{96}$ of work;
$\therefore$ B does the work in $\frac{96}{3}$ days $= 32$ days.

297. 30 children + 9 children + 1 child earn \$34 ;
∵ 40 children earn \$34 ;
1 child earns $\frac{34}{40}$;
∴ (36+6+5) children earn $\$\frac{47\times34}{40} = \$39.95$.

298. Cost price = $\frac{100}{106}$ of \$132.50 = \$125 ;
∴ loss per cent. = $\frac{100\times10}{125} = 8$.

299. Amount of stock bought = $\$\frac{6825\times100}{91}$
= \$7500.
Proceeds of first sale = $\$\frac{5000\times93\frac{3}{4}}{100}$
= \$4675.
" second sale = $\$\frac{2500\times85}{100}$
= \$2125.
Dividend from M. B. S. = $\$\frac{6806\times12}{175}$
= $\$466\frac{2}{7}$.
Original income = $\$\frac{6825\times6}{91}$
= \$450 ;
∴ increase = $\$16\frac{2}{7}$.

300. Area of room = $(14\frac{1}{4}\times13\frac{1}{3})$ sq. ft.
Area of 1 plank = $(\frac{8}{12}\times10)$ sq. ft. ;
∴ number of planks = $\frac{14\frac{1}{4}\times13\frac{1}{3}}{\frac{8}{12}\times10} = 28\frac{1}{2}$.
Number of c. in. in 1 plank = $(\frac{1}{2}\times8\times120)$ c. in. ;
∴ weight of $28\frac{1}{2}$ planks = $(28\frac{1}{2}\times\frac{1}{2}\times8\times120\times\frac{1}{2})$ oz.
= 6840 oz. = $427\frac{1}{2}$ lb.

301.
```
4957.5681 ( 70.41
49
-----
1404 | 5756
       5616
-----
14081 | 14081
        14081
-----
```

$\frac{129.4947}{60.75} = \frac{14.3883}{6.75} = \frac{1.5987}{.75}$
$= \frac{.5329}{.25}$, and the square root of this fraction is $\frac{.73}{.5}$
= 1·46.

302. Interest on \$157.50 for 5 yr. $= \$(189 - 157.50)$
$= \$31.50$;
$\therefore$ " \$100 for 1 yr. $= \$\frac{100 \times 31.50}{157.50 \times 5}$
$= \$4.$

303. Time from 2nd to 12th July $= 10$ days.
Interest on \$273.75 for $\frac{10}{365}$ yr. $= \$\frac{273.75 \times 10 \times 5}{365 \times 100}$
$= \$.375$;
$\therefore$ value of first bill $= \$(273.75 + .375)$
$= \$274.125.$
Time from 12th to 22nd July $= 10$ days.
Present value of \$456.875 $= \$\frac{456.875 \times 100}{100\frac{10}{73}}$
$= \$456.25.$

304. Suppose the cask to contain 12 gallons, of which 9 are wine and 3 water,
then $9 - \frac{3}{4}$ of part drawn = quantity of wine remaining;
and $3 - \frac{1}{4}$ of part drawn = quantity of water remaining;
$\therefore$ $3 + \frac{3}{4}$ of part drawn = quantity of water in the vessel when water is substituted for the part drawn off;
$\therefore$ $9 - \frac{3}{4}$ of part drawn $= 3 + \frac{3}{4}$ of part drawn;
$\therefore$ $\frac{6}{4}$ of part drawn $= 6$;
$\therefore$ part drawn $= 4$ gallons, that is, one-third of the mixture.

305. Amount of bread each eats $= \frac{8}{3}$ loaves;
$\therefore$ " " given by first $= \frac{7}{3}$ "
and " " given by second $= \frac{1}{3}$ "
$\therefore$ he pays the first 7 half-pence and the second 1 half-penny.

306. Interest of \$100 $= \$(100 \times \frac{3}{4} \times \frac{15}{200}) = \$5$;
$\therefore$ the bill of which \$5 is the discount $= \$105$;
$\therefore$ " \$48.75 " $= \$\frac{48.75 \times 105}{5}$
$= \$1023.75.$

307. Capital at end of 1st year
$= \frac{120}{100}$ of original capital.
" " 2nd year
$= \frac{137\frac{1}{2}}{100}$ of $\frac{120}{100}$ of "
" " 3rd year
$= \frac{60}{100}$ of $\frac{137\frac{1}{2}}{100}$ of $\frac{120}{100}$ of "
$= \frac{99}{100}$ of original capital;
original capital $- \frac{99}{100}$ of original capital $= \$200$;
$\therefore$ original capital $= 100 \times \$200 = \$20000$.

308. Cost of 1st horse $= \$\frac{100 \times 100}{125} = \$80$.
" 2nd " $= \$\frac{100 \times 100}{75} = \$133\frac{1}{3}$;
$\therefore$ he loses $\$(80 + 133\frac{1}{3} - 200) = \$13\frac{1}{3}$.

309. \$2 is the interest of the discount for 6 mo.
\$2 is also the interest of \$100 for 6 mo.;
$\therefore$ discount $= \$100$, and interest $= \$102$;
$\therefore$ the sum $= \$\frac{102 \times 100}{2} = \$5100$.

310. Area of 5 external sides
$= \{(54 + 44) \times 6\frac{1}{2} + 27 \times 22\}$ sq. ft. $= 1231$ sq. ft.,
and neglecting the thickness of the material the area of the inside will also be 1231 sq. ft.;
$\therefore$ cost $= \frac{2 \times 1231 \times 4\frac{1}{2}}{9}$ cts. $= \$12.31$.

311. (α) $3 - \dfrac{1}{2 - \frac{5}{31}}$ divided by $1 + \dfrac{1}{4 + \dfrac{1}{3 - \frac{2}{7}}}$

$= 3 - \frac{31}{57}$ divided by $1 + \dfrac{1}{4 + \frac{7}{19}}$
$= \frac{140}{57} \div (1 + \frac{19}{83}) = \frac{140}{57} \div \frac{102}{83} = \frac{5810}{2907} = 1\frac{2903}{2907}$.

(β) $\dfrac{7\frac{5}{6} - 1\frac{3}{6}}{6\frac{1}{2} - 3\frac{1}{2}} = \frac{5\frac{2}{3}}{3} = \frac{17}{9} = 1\frac{8}{9}$.

312. $\frac{3}{4}$ of selling price $= \frac{80}{100}$ of cost price ;
$\therefore$ $15 \times$ selling price $= 16 \times$ cost price ;
$\therefore$ selling price $= \frac{16}{15}$ of cost price ;
$\therefore$ the gain $= \frac{1}{15}$ of the cost price ;
$\therefore$ the gain per cent. $= \frac{100}{15} = 6\frac{2}{3}$.

313. \$175 has for principal \$100
$\therefore$ \$787.50 " $\$\frac{787.50 \times 100}{175}$
$= \$450.$
Time for which \$33.75 is interest on \$450 $= 1$ yr. ;
$\therefore$ " " \$540 " \$450 $= \frac{540}{33.75}$ yr.
$= 16$ yr.

314. In 4 years I save $4 \times 6\frac{2}{3}\%$, or $26\frac{2}{3}$ % of my income ;
$\therefore$ $(26\frac{2}{3}\% - 25\%)$ of my income $= \$50$;
$\therefore$ $\frac{11}{100}$ " $= \$50$;
$\therefore$ my income $= \$\frac{300 \times 50}{5}$
$= \$3000.$

315. $\$\frac{13}{1000}$ = tax from \$1 ;
$\therefore$ \$101.40 = " $\$\frac{101.40 \times 1000}{13}$
$= \$7800.$

316. Cost price of 1 lb. $= \frac{72 \times 100}{108} = 66\frac{2}{3}$ cents ;
$\therefore$ " 225 lb. $= 225 \times 66\frac{2}{3}$ cts. $= \$150.$
Sum to be realized $= \$\frac{150 \times 110}{100} = \$165.$
Selling price of 45 lb. $= 45 \times 72$ cents $= \$32.40$;
$\therefore$ " $(125 - 45)$ lb. $= \$132.60$;
$\therefore$ " of 1 lb. $= \$\frac{132.60}{180} = 73\frac{2}{3}$ cents.

317. Interest on \$500 $= \$(500 \times \frac{1}{2} \times \frac{8}{100}) = \$13\frac{1}{3}$.
\$1500 remain to be paid, and I ought to keep this sum beyond the original time of payment till the interest on it amounts to $\$13\frac{1}{3}$.

$\$120$ = the interest on it for 12 mo.;

$\therefore$ $\$13\frac{1}{3}$ = " " $\frac{13\frac{1}{3}\times 12}{120}$ mo.

$= 1\frac{1}{3}$ mo.

318. A runs 880 yd. while B runs 800 yd.;

then B runs 960 yd. while A runs $\frac{9}{10}$ of 960 yd., or, 864 yd.;

$\therefore$ B runs 1760 yd. while A runs (880 + 864) yd.

$= 1744$ yd.;

$\therefore$ B wins by 16 yd.

319. Cost of 1 egg in one case $\frac{1}{21}s.$

" " other case $\frac{1}{19}s.$;

$\therefore$ average cost of 1 egg $= \frac{\frac{1}{21}+\frac{1}{19}}{2}s. = \frac{20}{399}s.$

$\therefore$ selling price of 1 egg $= \frac{1}{20}s.$;

$\therefore$ sum lost on 1 egg $= (\frac{20}{399} - \frac{1}{20})s. = \frac{1}{7980}s.$;

$\therefore$ loss per cent. $= \frac{100\times\frac{1}{7980}}{\frac{20}{399}} = \frac{1}{4}.$

320. $(6\times\text{breadth})\times 11 =$ area to be papered;

$\therefore$ $66\times\text{breadth} = 143\times 3\times 2$ sq. ft.;

$\therefore$ breadth $= \frac{143\times 3\times 2}{66}$ ft. $= 13$ ft.;

$\therefore$ circumference of room $= 6\times 13$ ft $= 26$ yd.

321. $\frac{\frac{5}{6}+\frac{11}{45}-\frac{11}{18}}{\frac{43}{7}\times\frac{7}{2}-\frac{7\times 7\times 9}{3\times 2\times 7}+\frac{8}{5}} = \frac{7}{\frac{63}{5}} = \frac{1}{2\frac{7}{7}}$;

$\frac{40}{55}\times\frac{11}{7}\times\frac{21}{4}+\frac{1}{3}+\frac{1}{2\frac{7}{7}} = 6+\frac{1}{3}+\frac{1}{2\frac{7}{7}} = 6\frac{26}{27}$;

$\therefore$ $6\frac{26}{27}+\frac{1}{2\frac{7}{7}} = 7.$

322. $(\frac{6}{1000}$ of $500d. + \frac{420}{990}$ of $66s.)\times\frac{60}{11}$

$= (£11\ 8s.\ 3d.)\times\frac{60}{11} = £62\ 5s.$

323. From 9 a.m. on Monday to 2 p.m. on Friday there are $4\frac{5}{24}$ days.

Difference between the watches for 1 da. $= 3\frac{1}{2}$ min. ;

" " " $4\frac{5}{24}$ da.

$= 4\frac{5}{24} \times 3\frac{1}{2}$ min.

$= 14$ min. $13\frac{3}{4}$ sec.

324. 6 men reap 35 a. in 7×12 hr. ;

$\therefore$ 1 man reaps 1 a. in $\frac{6 \times 7 \times 12}{35}$ hr. ;

$\therefore$ 9 men reap 45 a. in $\frac{45 \times 6 \times 7 \times 12}{9 \times 35}$ hr. $= 72$ hr.

$\therefore$ they will take 9 da. of 8 hr. each.

325. Cost price of tea sold $= \frac{72 \times 100}{120}$ cts. $= 60$cts. ;

$\therefore$ he gains 12 cents on each lb. of 48 ct. tea and loses 6 cents on each lb. of 66 ct. tea ;

$\therefore$ he must put 1 lb. of the former to 2 lb. of the latter.

326. $\frac{1}{4}$ ct., or $\$\frac{1}{400}$ = difference of tax on \$1 ;

$\therefore$ \$3.60 = " " $\$\frac{3.60 \times 1}{\frac{1}{400}}$

$= \$1440.$

327. 5 cents in the \$ is paid by \$500 assets ;

$\therefore$ his debts $= \$\frac{100 \times 500}{5} = \$10000.$

and his assets $= \frac{40}{100}$ of \$10000 $= \$4000.$

328. Time 35 men do a work $= 38$ da. ;

$\therefore$ " 19 " " $= \frac{35 \times 38}{19}$ da.

$= 70$ days.

329. Robert's debt to Charles

$= \frac{3}{5}$ of Robert's debt to Charles $+ 10d.$;

$\therefore$ $\frac{1}{5}$ of Robert's debt to Charles $= 10d.$;

$\therefore$ " " " $= 3 \times 10d.$

$= 2s.\ 6d.$

330. Area of surface

$= 2 \times (4 \times 2\frac{1}{2} + 4 \times 3 + 3 \times 2\frac{1}{2})$ sq. ft.

$= 59$ sq. ft. ;

$\therefore$ cost $= \frac{59 \times 15}{9} d. = 8s.\ 2\frac{1}{3}d.$

331. $\frac{34}{9} \times \frac{18}{17} + 4\frac{1}{12} - 3\frac{9}{16} = 4 + 4\frac{1}{12} - 3\frac{9}{16} = 4\frac{25}{48} = 5\frac{1}{16} = \frac{7}{8}$;

$5\frac{1}{9} - \frac{63 \times 20}{8 \times 567} + \frac{1}{3} = 5\frac{4}{9} - \frac{5}{18}$;

$\frac{11 \times 7}{3 \times 29} \times \frac{29 \times 39}{8 \times 7} - 17\frac{3}{4} = \frac{143}{8} - \frac{71}{4} = \frac{1}{8}$;

$\frac{7}{8} + \frac{1}{8} = 1$.

332. Time he takes to ride 1 way $= \frac{2\frac{1}{4}}{2}$ hr. $= 1\frac{1}{8}$ hr.;

$\therefore$ " " walk 1 way $= (3\frac{3}{4} - 1\frac{1}{8})$ hr.

$= 2\frac{1}{8}$ hr.;

$\therefore$ " " to walk both ways $= 5$ hr.

333. $\frac{\text{Distance in miles}}{4} =$ the certain time in hr. $+ \frac{1}{12}$;

$\frac{\text{distance in miles}}{5} =$ the certain time in hr. $- \frac{1}{6}$;

$\therefore \frac{\text{distance in miles}}{4} - \frac{1}{12} = \frac{\text{distance in miles}}{5} + \frac{1}{6}$;

$\therefore \frac{\text{distance in miles}}{4} - \frac{\text{distance in miles}}{5} = \frac{1}{12} + \frac{1}{6}$;

$\therefore \frac{\text{distance in miles}}{20} = \frac{1}{4}$;

$\therefore$ distance $= 5$ miles.

334. A and B, and A and C contribute \$(1390 + 1500), or \$2890.

B and C contribute \$1590;

$\therefore$ twice A's contribution $=$ \$1300;

hence A contributes \$650, B \$740, C \$850, D \$960.

Now \$(650 + 740 + 850 + 960) gain \$1152;

$\therefore$ A's share $= \$\frac{650 \times 1152}{3200} =$ \$234,

B's " $= \$\frac{740 \times 1152}{3200} =$ \$266.40,

C's " $= \$\frac{850 \times 1152}{3200} =$ \$306,

D's " $= \$\frac{960 \times 1152}{3200} =$ \$345.60.

335. From A to B it takes $\frac{1}{2}$ of 7 hr., or $3\frac{1}{2}$ hr. ;

$\therefore$ from B to A it takes $5\frac{1}{4}$ hr. $- 3\frac{1}{2}$ hr., or $1\frac{3}{4}$ hr. ;

$\therefore$ from C to A it takes $2 \times 1\frac{3}{4}$ hr., or $3\frac{1}{2}$ hr.

336. $40 \times$ (number of 10 ct. pieces $+ \frac{5}{10}$) $=$ no. of nuts ;

$50 \times$ (number of 10 ct. pieces $- 1$) $=$ no. of nuts ;

$\therefore$ $40 \times$ no. of 10 ct. pieces $+ 20 = 50 \times$ no. of 10 ct. pieces $- 50$;

$\therefore$ $10 \times$ number of 10 ct. pieces $= 70$;

$\therefore$ number of 10 ct. pieces $= 7$;

$\therefore$ I have 7×10 ct. $= 70$ ct.

337. Income from 1st investment $= \$\frac{38940 \times 6}{99}$

$= \$2360.$

" 2nd " $= \$\frac{38940 \times 7}{118}$

$= \$2310$;

$\therefore$ the former is better by \$50.

338. The note is due on 21st Nov.

Number of days between 18 Aug. and 21 Nov. $= 95$

Interest on \$100 for 95 days $= \$\frac{285}{146}$;

$\therefore$ he gets $\$(100 - \frac{285}{146})$ from a note for \$100 ;

$\therefore$ " \$14315 from a note for $\$\frac{14315 \times 100}{\frac{14315}{146}}$

$= \$14600.$

339. 133 oxen to 20 a. $= 26\frac{3}{5}$ oxen to 4 a.

28 oxen to 5 a. $= 22\frac{2}{5}$ oxen to 4 a.

$26\frac{3}{5}$ oxen eat the original grass and 13 days' growth in 13 days ;

$\therefore$ 1 ox eats $\frac{\text{original grass} + 13 \text{ days' growth}}{13 \times 26\frac{3}{5}}$ in 1 day.

$22\frac{2}{5}$ oxen eat the original grass and 16 days' growth in 16 days ;

$\therefore$ 1 ox eats $\frac{\text{original grass} + 16 \text{ days' growth}}{16 \times 22\frac{2}{5}}$ in 1 day;

$$\therefore \frac{\text{original grass} + 13 \text{ days' growth}}{13 \times 26\frac{2}{5}} = \frac{\text{original grass} + 16 \text{ days' growth}}{16 \times 22\frac{2}{5}};$$

$\therefore$ 63 $\times$ original grass $=$ 4368 days' growth;

$\therefore$ original grass $= 69\frac{1}{3}$ days' growth.

Quantity of grass to be eaten $= (69\frac{1}{3} + 14)$ days' growth.

Quantity eaten by 1 ox in 14 days $= 14\left(\frac{69\frac{1}{3} + 16}{16 \times 22\frac{2}{5}}\right)$ days' growth;

$$\therefore \text{number of oxen required} = \frac{83\frac{1}{3}}{\frac{14(69\frac{1}{3}+16)}{16 \times 22\frac{2}{5}}} = \frac{83\frac{1}{3} \times 16 \times 22\frac{2}{5}}{14 \times 85\frac{1}{3}} = 25.$$

The following is another solution :—

339. 133 oxen in 13 days eat grass on 20 a. + growth on 20 a. for 13 days;

$\therefore$ 1 ox in 1 day eats grass on $\frac{20}{13 \times 133}$ a. + growth $\frac{13 \times 20}{13 \times 133}$ a. for 1 day.

Again,

28 oxen in 16 days eat grass on 5 a. + growth on 5 a. for 16 days;

$\therefore$ ox in 1 day eats grass on $\frac{5}{16 \times 28}$ a. + growth on $\frac{16 \times 5}{16 \times 28}$ a. for 1 day.

Hence,

$\frac{20}{13 \times 133}$ a. + $\frac{20}{133}$ a. growth $= \frac{5}{16 \times 28}$ a. + $\frac{5}{28}$ a. growth;

$\therefore \frac{20}{13 \times 133}$ a. $- \frac{5}{16 \times 28}$ a. $= \frac{5}{28}$ a. growth $- \frac{20}{133}$ a. growth;

$\therefore$ 3 a. $=$ 208 a. growth in 1 day.

hence, 1 a. growth in 1 day $= \frac{3}{208}$ a.

Hence, 133 oxen in 13 days eat grass on 20 a. + grass on $13 \times 20 \times \frac{3}{20\,8}$ a., or $23\frac{3}{4}$ a.; and it is required to find how many oxen in 14 days can eat the grass on 4 a. $+ 4 \times 14 \times \frac{3}{20\,8}$ a., or $4\frac{2}{2}\frac{1}{6}$ a.

Oxen which eat $23\frac{3}{4}$ a. in 13 days $= 133$

" " 1 " 1 " $= \frac{13 \times 133}{23\frac{3}{4}}$

" " $4\frac{2}{2}\frac{1}{6}$ " 14 " $= \frac{4\frac{2}{2}\frac{1}{6} \times 13 \times 133}{14 \times 23\frac{3}{4}}$

$= 25.$

340. $\frac{1}{2}$ of the constituency vote for A;

$\frac{1}{4}$ of the constituency vote for B;

$\frac{2}{3}$ of $\frac{1}{4}$ of the constituency vote for D and E;

now $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} = \frac{11}{12}$;

$\therefore \frac{1}{12}$ of the constituency $= 540$, and $\therefore$ number of electors $= 6480$.

A's votes are $\frac{6480}{2}$, or, 3240;

B's votes are $\frac{4}{10}$ of 3240 $+ \frac{1}{2}$ of 3240, or, 2916;

C's votes are $\frac{3}{10}$ of 3240 $+ \frac{3}{10}$ of 3240, or, 1944;

D's votes are $\frac{2}{10}$ of 3240 $+ \frac{1}{10}$ of 3240 $+ \frac{2}{3}$ of 1620, or, 2052;

E's votes are $\frac{1}{10}$ of 3240 $+ \frac{1}{10}$ of 3240 $+ \frac{2}{3}$ of 1620, or, 1728.

341. $\frac{3}{2} \times \frac{14}{5} + \frac{55 \times 4}{8 \times 11} + 5\frac{1}{2} + \frac{\cdot 773}{1\cdot 5} = \frac{21}{5} + \frac{5}{2} + \frac{11}{2} + \frac{696 \times 9}{906 \times 14}$

$= 12\frac{1}{5} + \frac{87}{175} = 12\frac{122}{175}.$

342. $\frac{\frac{45}{8} \times \frac{3}{2}}{\frac{6 \times 5}{5 \times 9} \times \frac{3}{31}} \times \frac{2}{5} \times \frac{\frac{3 \times 37}{2 \times 9}}{\frac{111}{8} \times \frac{16}{3}}$

$= \frac{45 \times 3 \times 31}{8 \times 2 \times 2} \times \frac{2 \times 37 \times 8}{5 \times 2 \times 111 \times 16} = 4\frac{23}{64}.$

343. Sum got for $\$104\frac{3}{4} = \$100$;

$\therefore$ " $\$2304.50 = \$\frac{2304.50 \times 100}{104\frac{3}{4}}$

$= \$2200.$

341. Price of stock $= \$\frac{9450\times5\frac{1}{2}}{787.50} = \$66.$

345 Part B fills in 1 min. $= \frac{1}{30}$.

" " $8\frac{2}{5}$ min. $= 8\frac{2}{5}\times\frac{1}{30} = \frac{7}{25}$;

$\therefore$ part A fills $= \frac{18}{25}$;

$\therefore$ time required $= (\frac{18}{25}\div\frac{1}{25})$ min.

$= 18$ min.

346. In the last 4 years he saves £200 + £120, or £320;

$\therefore$ his income $-\frac{9}{10}$ (his income + £40) $=$ £80;

$\therefore$ $\frac{1}{10}$ of his income $-$ £36 $=$ £80;

$\therefore$ his income $= 10\times($£36 + £80)

$=$ £1160.

347. 15 men and 30 children get £(177 − 60) = £117;

$\therefore$ 1 man and 2 children get £$\frac{117}{15}$ = £7 16s.

But 1 man and 1 child get £6;

$\therefore$ 1 child gets £1 16s.,

and 1 man gets £6 − £1 16s. = £4 4s.,

and 1 woman gets £3.

348. 1 kilogramme = weight of $\frac{1}{1000}$ cub. met. of water

= weight of $\frac{(\frac{5}{8000})^3}{1000}$ cub. miles of water

= weight of $\frac{(\frac{5}{8000}\times1760\times\frac{3}{6})^3}{1000}$ cub. fathoms of water

$= \frac{(\frac{5\times11}{100})^3\times6\times20\times112}{1000}$ lb. avoirdupois

$= \frac{27951}{12500}$ lb. avoirdupois;

$\therefore$ the ratio is 27951 : 12500.

349. 4285 − (2540 + 980) = 765, the number of grains of soda and potash that take up 980 grains of the sulphuric acid;

hence $\frac{49\times\text{number of gr. of soda}}{32}$

$+\frac{49\times(765-\text{number of gr. of soda})}{48} = 980;$

or, $3 \times$ number of gr. of soda $+ 2 \times (765 -$ number of gr. of soda$) = 20 \times 96$;

$\therefore$ number of gr. of soda $= 1920 - 1530 = 390$;

$\therefore$ number of gr. of potash $= 765 - 390 = 375$.

350. Area of sides $= \{ (42+31) \times 10 \}$ sq. ft.
$= 730$ sq. ft.;

area of windows $= (3 \times 8\frac{1}{3} \times 4\frac{1}{2})$ sq. ft.
$= 112\frac{1}{2}$ sq. ft.;

area of doors $= (4 \times 6\frac{1}{2} \times 3\frac{1}{4})$ sq. ft.
$= 84\frac{1}{2}$ sq. ft.;

area of fireplace $= (6 \times 4)$ sq. ft. $= 24$ sq. ft.;

area of skirting $= (54 \times 1\frac{3}{4})$ sq. ft. $= 90$ sq. ft.;

$\therefore$ area to be papered $= (730 - 311)$ sq. ft.
$= 419$ sq. ft.;

$\therefore$ cost $= 419 \times 5$ cts. $= \$20.95$.

APPENDIX.

I.—Page 330.

1. Art. 1.

2. Arts. 2 and 3.

3. Let x be the required sum; then if r be the rate of interest, we shall have

$$M = PR^t$$

$$\text{and } P = xR^t;$$

$$\text{whence } \frac{P}{M} = \frac{x}{P};$$

$$\therefore x = \frac{P^2}{M}.$$

4. Here, we have

$2P = P(1+r)^n$, and $\therefore 2 = (1+r)^n$.

Also, $2P = P(1+2r)^m$, and $\therefore 2 = (1+2r)^m$;

whence, $(1+2r)^m = (1+r)^n$;

$$\therefore\ m \log (1+2r) = n \log (1+r);$$

$$\therefore\ \frac{m}{n} = \frac{\log(1+r)}{\log(1+2r)}$$

$$> \frac{\log(1+r)}{\log(1+2r+r^2)}$$

$$> \frac{\log(1+r)}{\log(1+r)^2}$$

$$> \tfrac{1}{2}.$$

5. Let P denote the sum of money, and if n be the required number of years, we shall have

$$3\mathrm{P} = \mathrm{P}(1.05)^n;$$

whence $(1.05)^n = 3$;

$$\therefore\ n \log (1.05) = \log 3;$$

$$\therefore\ n = \frac{\log 3}{\log (1.05)}$$

$$= \frac{.4771212}{.0211893}$$

$$= 22.5 \text{ years.}$$

6. Let x, y, z denote the three shares; then we shall have

$$x + y + z = \mathrm{P};$$

also, $x\mathrm{R}^a = y\mathrm{R}^b = z\mathrm{R}^c$, are the equations of condition;

whence $y = \mathrm{R}^{a-b}x$, and $z = \mathrm{R}^{a-c}x$; so that

$$x + \mathrm{R}^{a-b}x + \mathrm{R}^{a-c}x = \mathrm{P};$$

whence $x\mathrm{R}^{b+c} + x\mathrm{R}^{a+c} + x\mathrm{R}^{a+b} = \mathrm{PR}^{b+c}$;

$$\text{and } \therefore\ x = \frac{\mathrm{PR}^{b+c}}{\mathrm{R}^{b+c} + \mathrm{R}^{a+c} + \mathrm{R}^{a+b}};$$

$$\text{similarly, } y = \frac{\mathrm{PR}^{c+a}}{\mathrm{R}^{c+a} + \mathrm{R}^{b+a} + \mathrm{R}^{b+c}},$$

$$\text{and } z = \frac{\mathrm{PR}^{a+b}}{\mathrm{R}^{a+b} + \mathrm{R}^{c+b} + \mathrm{R}^{c+a}}.$$

7. Let r be the interest of \$1 for 1 year; then amount of \$4410 for 2 yr. S. I. $= 4410(1+2r)$; and,

" \$4400 " C. I. $= 4400(1+r)^2$;

whence $4410(1+2r) = 4400(1+r)^2$,

or, $440r^2 - 2r = 1$;

$\therefore r = \frac{1}{20}$ or 5 per cent.

8. Let P represent the population; then, population at end of nth year

$$= P\left\{1 + \frac{n-m}{mn}\right\}^n; \text{ see Ex. 4;}$$

therefore, by the question, we have

$$P\left\{1 + \frac{n-m}{mn}\right\}^n = 2P,$$

$$\text{or, } n \log \left\{1 + \frac{n-m}{mn}\right\} = \log 2;$$

$$\therefore n = \frac{\log 2}{\log(mn+n-m) - \log mn}.$$

9. Let P_n represent his property at the end of n yr.; in the next year, the $(n+1)$th, his interest $= P_n r$,

and expenditure $= (n+1)m.P_n r$;

$\therefore$ the property left

$$= P_n + P_n r - (n+1)m.P_n r$$

$$= P_n.\{1 + r - (n+1)mr\}.$$

Now putting $2p$ for $n+1$, or $2p-1$ for n, we have

$$P_{2p-1}.\{1+r-2pmr\} = 0;$$

$$\therefore 1+r = 2pmr.$$

But his expenditure in the pth year $= pmP_{p-1}r$,

and property left at end of pth year $= P_{p-1}\{1+r-pmr\}$

$= pmP_{p-1}r$,

(since $1+r = 2pmr$) = expenditure in pth years.

10. From Ex. 3, we see that

$$M = Pe^{nr};$$

by the question

$$6 = e^{n_{\frac{1}{100}}};$$

$$\text{or } 6^{100} = e^{n};$$

$$\therefore 100 \log 6 = n \log e;$$

$$\therefore n = \frac{100 \log 6}{\log e}.$$

11. Let P denote his capital; r the interest of \$1 for one year.

Then the sum he spends every year is $2Pr$.

At the end of the first year he has left $P(1+r) - 2Pr$ or $P - Pr$.

At the end of the second year $(P - Pr)(1+r) - 2Pr$ or $P - 2Pr - Pr^2$.

At the end of the third year $(P - 2Pr - Pr^2)(1+r) - 2Pr$ or $P - 3Pr - 3Pr^2 - Pr^3$.

By proceeding in this way we may show that the sum he has left at the end of n years is

$$P - nPr - \frac{n(n-1)}{1 \cdot 2} Pr^2 - \ldots - Pr^n \text{ or } 2P - P(1+r)^n.$$

Thus we have to find n from the equation

$$2P - P(1+r)^n = 0, \text{ or } (1+r)^n = 2.$$

Putting for r its value $\frac{4}{100}$ we get $(\frac{104}{100})^n = 2$.

Taking the logarithm of each side

$$n(\log. 13 + \log. 8 - \log 100) = \log. 2;$$

$$\therefore n = \frac{.301030}{.017034} = 17.673, \text{ nearly}$$

12. Births 62 in 1000
Deaths 27 " 1000

Increase 35 " 1000 or $3\frac{1}{2}$ per cent.

Population at end of 5 years $= 35743\left(1+\frac{3\frac{1}{2}}{100}\right)^5$

$= 35743\ (1{\cdot}035)^5$

$= 42451{\cdot}471\ldots$

Hence, increase $= 42451{\cdot}471 - 35743$

$= 6708{\cdot}471\ldots$

13. The annual increase $= \frac{P}{45}$.

" decrease $= \frac{P}{60}$;

$\therefore$ the net annual increase $= \frac{P}{45} - \frac{P}{60} = \frac{P}{180}$.

Hence, by the question,

$$P\left\{1+\tfrac{1}{180}\right\}^n = 2P;$$

$$\therefore \left(\tfrac{181}{180}\right)^n = 2,$$

or, $n\{\log 181 - \log 180\} = \log 2$;

$$\therefore n = \frac{\cdot 30103}{\cdot 00249074}$$

$= 125$ nearly.

14. Let P denote the sum borrowed.

Then $\frac{P}{20}$ = annual income in the first case; and

$\frac{P-600}{25}$ = " " second " ;

whence, by the question,

$$\frac{P-600}{25} = \frac{2}{3}\cdot\frac{P}{20} = \frac{P}{30};$$

$$\therefore P = \$3600.$$

15. If r denote the interest of \$1 for one year,

amount of debt in n years $= a\mathrm{R}^n$.

Amount of annual payments

$$= \frac{a}{m}\left\{\mathrm{R}^{n-1}+\mathrm{R}^{n-2}+\ldots+1\right\} = \frac{a}{m}\left\{\frac{\mathrm{R}^n-1}{\mathrm{R}-1}\right\};$$

by the question, these two amounts must be equal; hence, we have

$$a(1+r)^n = \frac{a}{mr}\left\{(1+r)^n - 1\right\};$$

$$\therefore\ (1-mr)\,(1+r)^n = 1.$$

II.—Page 335.

1. Bank Discount at 5 % $= \frac{1}{20}\mathrm{P} = \$37.10$;

$$\therefore\ \mathrm{P} = \$742.$$

Present worth of \$742 $= \$\frac{742}{1+\frac{1}{20}}$

$= \$706.66\frac{2}{3}$.

2. Let S represent the sum of money; then

$$\frac{4}{100}\,\mathrm{S} = \$536.25;$$

$$\therefore\ \mathrm{S} = \$13406.25.$$

If P represent the face of the note,

$$\mathrm{V} = \frac{\mathrm{P}}{1+nr} = \frac{\mathrm{P}}{1+\frac{1}{3}\cdot\frac{4}{100}} = \frac{300\mathrm{P}}{304} = \$13406.25;$$

$$\therefore\ \mathrm{P} = \$13585.$$

3. If P represent the sum, then

$$4 \text{ per cent.} = \frac{\mathrm{P}}{25}, \text{ and}$$

$$\text{Discount} = \frac{\mathrm{P}}{26} = \$15;$$

$$\therefore\ \mathrm{P} = \$390.$$

Interest on \$390 at 5 % $= \frac{1}{20}$ of \$390

$= \$19.50$.

4. $$\frac{\mathrm{PI}}{\mathrm{P}+\mathrm{I}} = \mathrm{D};\quad \text{Art. 9.}$$

hence, $$\frac{180\mathrm{P}}{\mathrm{P}+180} = 150;$$

$$\therefore\ \frac{\mathrm{P}}{\mathrm{P}+180} = \frac{5}{6};$$

$$\therefore \frac{P}{180} = 5,$$

$$\text{or } P = \$900.$$

5. Interest on \$A for 1 yr. $= Ar$.

Discount on \$B " $= \frac{Br}{1+r}$;

Hence, $Ar = \frac{Br}{1+r}$;

$$A + Ar = B;$$

$$\therefore r = \frac{B-A}{A};$$

$$\therefore 100r = 100.\frac{B-A}{A}.$$

6. 3 % for 12 mo. $= 1\frac{1}{4}$ % for 5 mo.;

$\therefore$ real value of stock $= 90 - 1\frac{1}{4} = 88\frac{3}{4}$.

Income from $88\frac{3}{4} = 3$;

" $100 = \frac{100 \times 3}{88\frac{3}{4}}$

$= 3\frac{27}{71}$.

7. Let x = the amount of the bill due from B to A; then,

Present Worth of \$$a$ due in m years $= aR^{-m}$,

" " \$$b$ " n " $= bR^{-n}$,

" " \$$x$ " p " $= xR^{-p}$;

whence, by the conditions of the question, we have

$$xR^{-p} + bR^{-n} = aR^{-m};$$

$$\therefore xR^{-p} = aR^{-m} - bR^{-n};$$

hence $x = aR^{p-m} - bR^{p-n}$.

8. Let P represent the sum;

then \$$a$ assures \$100;

$\therefore$ \$P " $\$\frac{P \times 100}{a}$.

A − P is the reduced income; whence, by condition of question

$$\begin{aligned} A - P &= \text{Int. on } \$\frac{P \times 100}{a}, \text{ at } r\ \%, \\ &= \frac{r}{100} \text{ of } \$\frac{P \times 100}{a} \\ &= \frac{rP}{a}; \\ P &= \frac{Aa}{a+r}. \end{aligned}$$

9. Let p = price of goods;

$\therefore \frac{9p}{10}$ = price paid in 6 months; and its present worth

$$\begin{aligned} &= \frac{9p}{10} \div \left(1 + \frac{2\frac{1}{2}}{100}\right) \\ &= \frac{36p}{41}. \end{aligned}$$

Again, let x = the *rate* of discount allowed on payment at *two* months, so that $p\ (1-x)$ is *then* paid for goods.

The present worth of $p\ (1-x)$ at 2 mos. $= \frac{120p(1-x)}{121}$;

hence, $\frac{120\,p\,(1-x)}{121} = \frac{36p}{41}$

$$\begin{aligned} \therefore x &= .11463\ldots \\ \text{and } 100x &= 11.463\ldots \text{ rate } \textit{per cent.} \end{aligned}$$

Or, we may more briefly reason thus:—

For every \$100, B pays \$90, if paid in 6 months; and present worth of \$90 = $\frac{100}{102\frac{1}{2}}$ of \$90 = \$87.8040. Amount of \$87.8048 for 2 mos. at 5 % = \$88.5365; $\therefore$ \$100 − \$88.5365 = \$11.463.

10. Since $D = \frac{PI}{P+I}$, Art. 9, we have

$$7.50 = \frac{100I}{100+I};$$

$$\therefore\ I = 8\tfrac{108}{999};$$

and if the interest in 1 year is $5\tfrac{405}{999}$, the time in which $8\tfrac{108}{999}$ will be the interest, the rate being the same, will be

$$8\tfrac{108}{999} \div 5\tfrac{405}{999} = 1\tfrac{1}{2} \text{ years.}$$

III.—Page 344.

1. Since

$$P = \frac{nA}{2} \cdot \frac{2+(n-1)r}{1+nr}, \text{ simple interest,}$$

We have, in this case

$$P = \$\left(\frac{5\times 530}{2} \cdot \frac{2+(5-1)\times .07}{1+5\times .07}\right)$$

$$= \$\left(\frac{5\times 530}{2} \cdot \frac{200+28}{135}\right)$$

$$= \$(530 \times \tfrac{38}{9})$$

$$= \$2237.77\ldots$$

Again, since

$$P = \frac{A}{r}\left\{1 - R^{-n}\right\}, \text{ Compound Interest}$$

$$= \frac{530}{.07}\left\{\frac{(1.07)^5 - 1}{(1.07)^5}\right\}$$

$$= \$2173.10.$$

2. $$10000 = \frac{A}{.05}\left\{1 - (1.05)^{-20}\right\};$$

$$\therefore\ 500 = A\left\{\frac{(1.05)^{20}-1}{(1.05)^{20}}\right\}$$

$$= A\left(\frac{1.6533}{2.6533}\right);$$

$$\therefore A = \$\frac{1326.65}{1.6533}$$
$$= \$802.42.$$

3. Since

$$P = \frac{A}{r}, \text{ Art. 19, in this case}$$
$$= \$\frac{1000}{.06}$$
$$= \$16666.66\tfrac{2}{3}.$$

4. Since

$$P = \frac{A}{R^{p+q}}\left\{\frac{R^q-1}{R-1}\right\}, \text{ Art. 20,}$$

in this case, we have

$$P = \frac{400}{(1{\cdot}06)^{15}}\left\{\frac{(1{\cdot}06)^{10}-1}{{\cdot}06}\right\}$$
$$= \$2199.95\ldots$$

5. Son's $= \frac{1000}{.06}\{1-(1{\cdot}06)^{-10}\}$
$$= \frac{1000}{.06}\left\{\frac{(1{\cdot}06)^{10}-1}{(1{\cdot}06)^{10}}\right\}$$
$$= \frac{1000}{.06}\left(\frac{.79085}{1.79085}\right)$$
$$= \$7360.08.$$

Daughter's $= \frac{1000}{(1{\cdot}06)^{30}}\left\{\frac{(1{\cdot}06)^{20}-1}{{\cdot}06}\right\}$
$$= \frac{1000}{5.7434887}\left\{\frac{3.20713-1}{.06}\right\}$$
$$= \$6404.74.$$

Institution's $= \frac{1000}{(1{\cdot}06)^{30}\times{\cdot}06}$
$$= \frac{1000}{5.743488\times.06}$$
$$= \$2901.83.$$

6. $$M = A.\frac{R^n - 1}{R - 1}$$
$$= \tfrac{100}{.06}\{(1{\cdot}06)^{18} - 1\}$$
$$= \tfrac{100}{.06}(1{\cdot}85434)$$
$$= \$3090.56.$$

7. We may consider the £3769 as the *Present Worth* of an Annuity that has 30 years to run, and, therefore,

$$£3769 = \frac{A}{.04}\left\{1 - (1.03)^{-30}\right\};$$
$$\therefore\ £150.76 = A\left\{1 - (1.04)^{-30}\right\};$$
$$\therefore\ A = \frac{£150.76}{1 - (1.04)^{-30}}$$

8. The lease is renewed for a years; $\$d$ may, therefore, be considered as the Present Worth of an Annuity that has a years to run. Hence,

dR^a = amount of annuity = M;

$$\text{but}\quad M = A.\frac{R^a - 1}{R - 1};$$
$$\therefore\quad dR^a = A.\frac{R^a - 1}{R - 1}$$
$$= \frac{A}{r}(R^a - 1);$$
$$\therefore\quad A = \frac{drR^a}{R^a - 1}.$$

9. The fine P may be regarded as the Present Worth of an Annuity, A or *extra-rent*;

$$\therefore\ P = \frac{A}{r}(1 - R^{-n});$$
$$\therefore\ A = \frac{Pr}{1 - R^{-n}}.$$

The new fine, f, must provide for this extra rent during the $q - p$ years, which are to be added to the original term = fine for q years — fine for p years;

$$\therefore f = \frac{A}{r}\left\{(1 - R^{-q}) - (1 - R^{-p})\right\}$$

$$= \frac{1}{r} \cdot \frac{Pr}{1 - R^{-n}}\left(R^{-p} - R^{-q}\right)$$

$$= \frac{P}{1 - R^{-n}}\left(R^{-p} - R^{-q}\right)$$

10. Each owns $\frac{a}{2}$.

Present Worth of a freehold producing $\$\frac{a}{2}$ per annum:

$$= \frac{a}{2} \cdot \frac{1}{r}$$

$$= \frac{a}{2r}.$$

Present Worth of an Annuity of $\$b$ to continue for n years

$$= \frac{b}{r}\left(1 - R^{-n}\right);$$

$$\therefore \frac{a}{2r} = \frac{b}{r}\left(1 - R^{-n}\right),$$

$$\text{or, } \frac{a}{b} = 2\left(1 - R^{-n}\right)$$

$$= 2\left(1 - \frac{1}{R^{n}}\right).$$

11. Since

$$P = \frac{A}{r}\left(1 - R^{-n}\right),$$

We have, in this case,

$$P = \frac{\frac{1}{n}}{\frac{r}{n}}\left\{1 - \left(1 + \frac{r}{n}\right)^{-mn}\right\}$$

$$= \frac{1}{r}\left\{1 - \left(1 + \frac{r}{n}\right)^{-mn}\right\}.$$

Again,

$$P = \frac{\frac{1}{n}}{\frac{r}{n}}\left\{1 - \left(1 + \frac{r}{n}\right)^{-mn}\right\}$$

$$= \frac{1}{r}\left\{1 - \left(1 - mn\frac{r}{n} + \frac{mn(mn+1)}{1\cdot 2}\left(\frac{r}{n}\right)^2 - \frac{mn(mn+1)(mn+2)}{1\cdot 2\cdot 3}\left(\frac{r}{n}\right)^3 + \&c.,\right)\right\}$$

$$= \frac{1}{r}\left\{mr - \frac{m(mn+1)}{n}\frac{r^2}{1\cdot 2} + \&c.\right\}$$

$$= \frac{1}{r}\left\{mr - \left(m^2 + \frac{m}{n}\right)\frac{r^2}{1\cdot 2} + \left(m^2 + \frac{m}{n}\right)\left(\frac{m}{2} + \frac{m}{n}\right)\frac{r^3}{2} - \&c.\right\}$$

Now, as n increases, it is plain that $\frac{m}{n}$ decreases, and tends to *zero* as its *limit;* hence, as n increases, the limit of the above series is

$$\frac{1}{r}\left(mr - \frac{m^2r^2}{1\cdot 2} + \frac{m^3r^3}{1\cdot 2\cdot 3} - \&c.\right)$$

$$= \frac{1}{r}\left\{1-\left(1-mr+\frac{m^2r^2}{1\cdot2}-\frac{m^3r^3}{1\cdot2\cdot3}+\&c.\right)\right\}$$

$$= \frac{1-e^{-mr}}{r}.$$

Hence, limit of $P = \frac{1-e^{-mr}}{r}$.

12. The Present Worth of an Annuity of \$10 per month for 6 months at $\frac{1}{2}$ % per month

$$= \frac{10}{.005}\left\{1-\left(1+\frac{5}{10^3}\right)^{-6}\right\}.$$

If P be the sum to be paid at once, P in 19 years must amount to the preceding present worth; hence, we have

$$P\left(1+\frac{5}{10^3}\right)^{19} = \frac{10}{.005}\left\{1-\left(1+\frac{5}{10^3}\right)^{-6}\right\}$$

Now $\left(1+\frac{5}{10^3}\right)^{19} = 1+\frac{19}{1}\cdot\frac{5}{10^3}+\frac{19.18}{1.2}\cdot\frac{5^2}{10^6}+\frac{19.18.17}{1.2.3}\cdot\frac{5^3}{10^9}+\&c.$

$$= 1+.095+.004275+.000121125+\&c.$$

$$= 1.099396\ldots$$

And $\frac{10}{.005}\left\{1-\left(1+\frac{5}{10^3}\right)^{-6}\right\}$

$$= 2000\left\{1-\left(1-\frac{6}{1}\cdot\frac{5}{10^3}+\frac{6.7}{1.2}\cdot\frac{5^2}{10^6}-\frac{6.7.8}{1.2.3}\cdot\frac{5^3}{10^9}+\frac{6.7.8.9}{1.2.3.4}\cdot\frac{5^4}{10^{12}}-\&c\right)\right\}$$

$$= 2000\ (.03-.000525+.000007-.00000007875+\&c.)$$

$$= 58\ 9638\ldots.$$

Hence P $(1.099396) = 58.963844$;

$$\therefore P = \$53.63.$$

13. By reference to Ex. 3, we see that

$$P(1{\cdot}05)^{3\,5\frac{1}{2}} = 4000 + \frac{100\left\{(1{\cdot}05)^{3\,6} - 1\right\}}{{\cdot}05}$$

$$(1{\cdot}05)^{3\,6} = 5{\cdot}7918149$$

$$(1{\cdot}05)^{3\,5\frac{1}{2}} = 5{\cdot}6064572.$$

Again, $4000 + \dfrac{100\left\{(1{\cdot}05)^{3\,6} - 1\right\}}{{\cdot}05}$

$$= 4000 + 2000 \times 4{\cdot}7918149$$

$$= 13583{\cdot}6298.$$

Hence, $P(5{\cdot}6064572) = 13583{\cdot}6298$;

$$\therefore\ P = \$2422.85.$$

14. (1) The equation established in Art. 11, which is the same as that given in the exercise, proves the first statement.

(2) Multiplying each side of the equation

$$s_1 R^{-t_1} + s_2 R^{-t_2} = \left(s_1 + s_2\right) R^{-t},$$

by R^T, where T is some time subsequent to t_2, we have

$$s_1 R^{T-t_1} + s_2 R^{T-t_2} = (s_1 + s_2) R^{T-t},$$

which shows that the amounts are the same at the subsequent time T.

(3) Again, multiplying each side of

$$s_1 R^{-t_1} + s_2 R^{-t_2} = (s_1 + s_2) R^{-t}$$

by R^t, and we have

$$s_1 R^{t-t_1} + s_2 R^{t-t_2} = (s_1 + s_2) R^0$$

$$= s_1 + s_2;$$

$$\therefore\ s_1 R^{t-t_1} - s_1 = s_2 - s_2 R^{-(t_2-t)}$$

Now, the *compound interest* of any sum is found by subtracting the sum from its amount for the given time; therefore, $s_1 R^{t-t_1} - s_1$ is the C. I. of s_1 for the time it is overdue.

The discount of any sum is found by subtracting the *Present Worth*, for the given time, from the sum itself; hence, $s_2 - s_2 R^{-(t_2-t)}$ is the discount on s_2 for the time before it is due; and the equation shows that "at the intermediate time, t, of payment, the interest of the sum overdue is the discount of that not due."

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